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FeyRay Evaluation: 2007

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14. ABSTRACT

The capabilities of the FeyRay acoustic prediction model (version 1.12 Foreman 2007) are analyzed through comparison with the Navy Standard Parabolic Equation Model (NSPE), and the GRAB (Gaussian Ray Bundle eigenray propagation model) in a limited number of environments. Measurements of the relationship between GRAB and NSPE are designed to quantify the meaning of our FeyRay measurements in terms of another commonly used OAML model for Gaussian Beam propagation. The measurements reported here indicate that the accuracy and execution speed of FeyRay are sufficient to justify its use in high-fidelity training systems.

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1.0 INTRODUCTION

Analysis of the prognostic capabilities of an acoustic predictive system is an involved task which, given resource constraints, require that the analysis of the system be limited and directed toward the model's intended applications. In this report, the capabilities of the FeyRay acoustic prediction model (version 1.12 Foreman 2007) will be analyzed through comparison with the Navy Standard Parabolic Equation Model (NSPE), and the GRAB (Gaussian Ray Bundle eigenray propagation model) in a limited number of environments. Measurements of the relationship between GRAB and NSPE are designed to quantify the meaning of our FeyRay measurements in terms of another commonly used OAML model for Gaussian Beam propagation.

Each of the models is exercised on system where the results are known to match the results of the standard output cases (for GRAB and NSPE), or that delivered by the developer (FeyRay). The FeyRay, GRAB, and PE executions were made on a Sun 1.8 GHz Opteron system running Debian 3.1 Linux, with the models compiling using the GCC compiler suite. Secondary runs, to ensure quality control, were made on a Apple Macintosh G4 computer using Absoft C++/Fortran95 compilers.

1.1 Scope

The acoustic analysis contained in this report is based on comparing the output of FeyRay and GRAB to an accepted acoustic benchmark, NSPE. Accepting NSPE as benchmark does not imply that the output of the NSPE is invariably correct, but recognizes NSPE as a reasonable standard to compare against. The imposed constraints limit the number of cases to be analyzed in any depth to a small number, in this case 7. But by analyzing a large number of cases in a strictly statistical manner some conclusions about the overall behavior of the FeyRay model with respect to its peers will also be obtained.

Each of the models was run in the mode indicated in the documentation to obtain optimal results. The ray fan selection in GRAB was set to -89° to 89° in 0.1° increments, while the beam fan selection was made internally by FeyRay. GRAB was executed using both version 2 and 3, with the optimal result selected.
In section 2 of this report the statistical metrics used to compare the model predictions are introduced. In section 3, the one dimensional sound speed fitting capabilities of FeyRay are summarized, the performance of FeyRay and GRAB are shown against the know Lloyd's mirror result, and the performance of the FeyRay and GRAB predictions against a number of NSPE results in randomly selected environments are shown. Section 4 shows the modeling capabilities in deep water and section 5 shows the capabilities in shallow water. Section 6 contains the conclusions.

2.0 METRICS

Measuring the difference between predictions made by different software systems can be done in a number of ways. The question that arises, or that should arise, is what is the natural way to measure the performance of a prediction system? A transmission loss prediction system predicts the relative acoustic energy as a function of range. In the field it is not possible to measure transmission loss directly. In fact, one measures received levels, then computes transmission losses from a calibrated source. As such, a sonar system operator does not detect transmission loss, rather, the operator detects a signal in excess of the background ambient level. That is, detected signals exceed some threshold level. As such, the measurement criteria of effectiveness of a prediction system should be based on the ability of the prediction system to predict ranges at which the transmission losses are low enough to allow detection.

A number of computational issues arise that must be dealt with on an individual basis. While each of these issues has effects on the predictive capability of a model, most are not seen when viewing the product of an acoustic simulation. Experience with the testing of other Gaussian ray models for OAML indicates that three factors are of critical importance:

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- the sound speed fitting capability of the model,
- accuracy of the model in the Lloyds mirror scenario, and
- the ability of the model to cope with a wide variety of input (robustness).

Thresholds were defined in terms of the Figure of Merit needed to detect three types of passive sonar targets:

- Strong Target = 100 dB
- Medium Target = 80 dB
- Weak Target = 60 dB

Transmission loss values below the 100 dB level were assumed to be tactically insignificant.

Three statistics are defined to measure the effectiveness of the performance prediction. First, the difference of each model from the reference solution is measured and averaged.

$$\Delta L = \frac{1}{N}\sum_{n=1}^{N} TL(r_n) - PE(r_n)$$

where:

r_n	= series of ranges at which TL is calculated.
TL	= transmission loss of model under investigation
PE	= transmission loss of NSPE model at the same range,
ΔL	= average difference in level (smaller is better).

In order to focus on tactically significant results, this calculation excludes any range where the NSPE solution dips below the 100 dB threshold. Measurements are made in decibel space rather than in linear units to measure the average effect of differences on signal excess and, in turn, sonar performance.

The second metric is a correlation error based on the Pearson product-moment correlation coefficient.

$$r_{error} = 1 - r = 1 - \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

where

r	= the Pearson product-moment correlation coefficient,
σ_{xy}	= the covariance of the sequences x and y,
σ_x	= the standard deviation of x,
σ_y	= the standard deviation of y, and
r_{error}	= the average correlation error (smaller is better).

The correlation error measures differences in the fluctuations between models. It is designed to provide a measure of model-to-model equivalence that is independent of the average error in level. Correlation error was chosen instead of correlation as a metric so that each metric specifies improved equivalence to the NSPE as values decrease. Measurements are made in decibel space rather than in linear units to measure the average effect of fluctuations on signal excess and, in turn, sonar performance.

The last metric is an estimate of detection range for the three types of targets specified above. Tabulated values will be presented which illustrate the ranges at which detection first ceases to occur, and last ceases to occur are accumulated.

3.0 COMPONENTS TESTS

These tests were designed to analyze individual features of the model.

3.1 Sound Speed Fitting

FeyRay requires that the sound speed interpolant, and its first 3 derivatives, be continuous. The FeyRay sound speed algorithm is a two part construction. The algorithm is a tension spline interpolant in the vertical (a global fit along a single sound speed profile), coupled with a degenerate firth order polynomial in the horizontal between two adjacent vertical profiles (a local method in the horizontal).

The fit of the interpolant to the input sound speed field was examined using a Munk profile (Munk 1974) using the parameters found in Porter and Bucker (1987). A source is placed at 1000m depth , a receiver at 800m depth, and beams are launched from -14.66 to 14.66 degrees with a spacing of 0.02 degrees, and 0.01 degrees. At each call in the ray tracing algorithm where the sound speed is updated the value of the depth and the sound speed is intercepted. Figure 1 shows a plot of the sound speed interpolant versus depth for the two different spacing. The two set of values define the Munk profile to a very high degree.

There are small differences between the values derived from the analytic Munk expression, and the FeyRay derived interpolant. Figure 2 shows a plot of the difference between the FeyRay interpolant and the true value. The interpolant varies from the true value by less than 0.001 per cent, which is sufficiently accurate for acoustic purposes. However, there is an aspect of the fit that must be mentioned. Since the fit oscillates between negative and positive variations, this implies that higher derivatives also oscillate. The magnitude of the oscillations will be small, but may be visible when the eigenrays from a fine input beam spacing are compared to the eigenrays from a coarse input beam spacing.

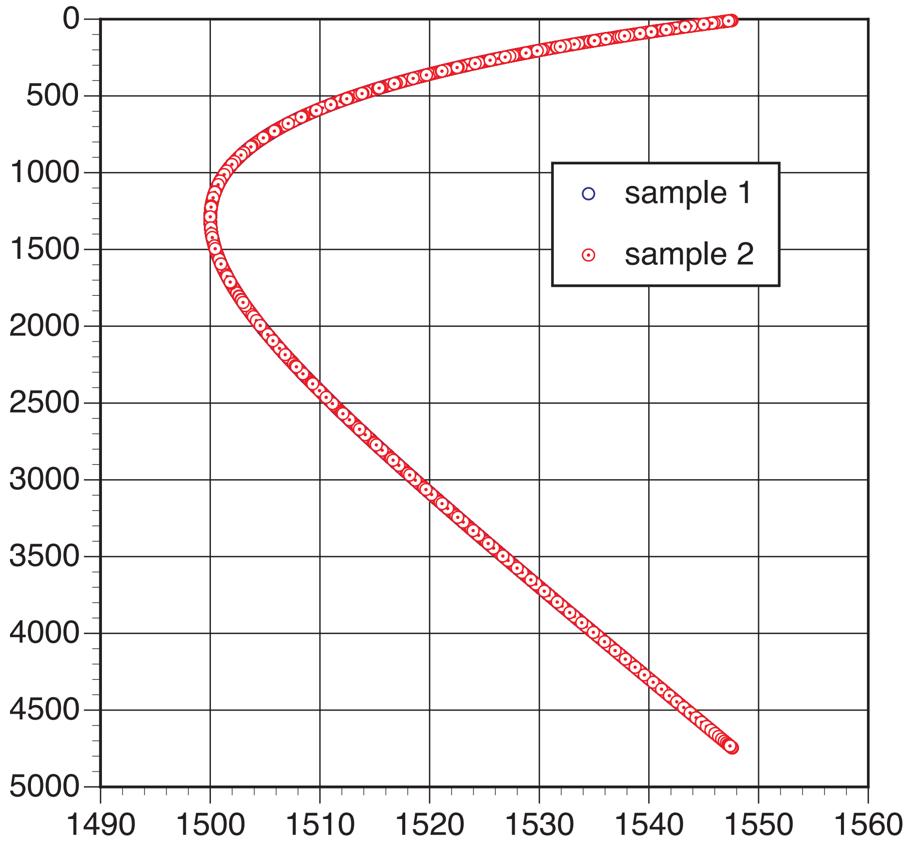

Figure 1. Plot of Munk sound speed obtained from FeyRay interpolation for two different beam fans.

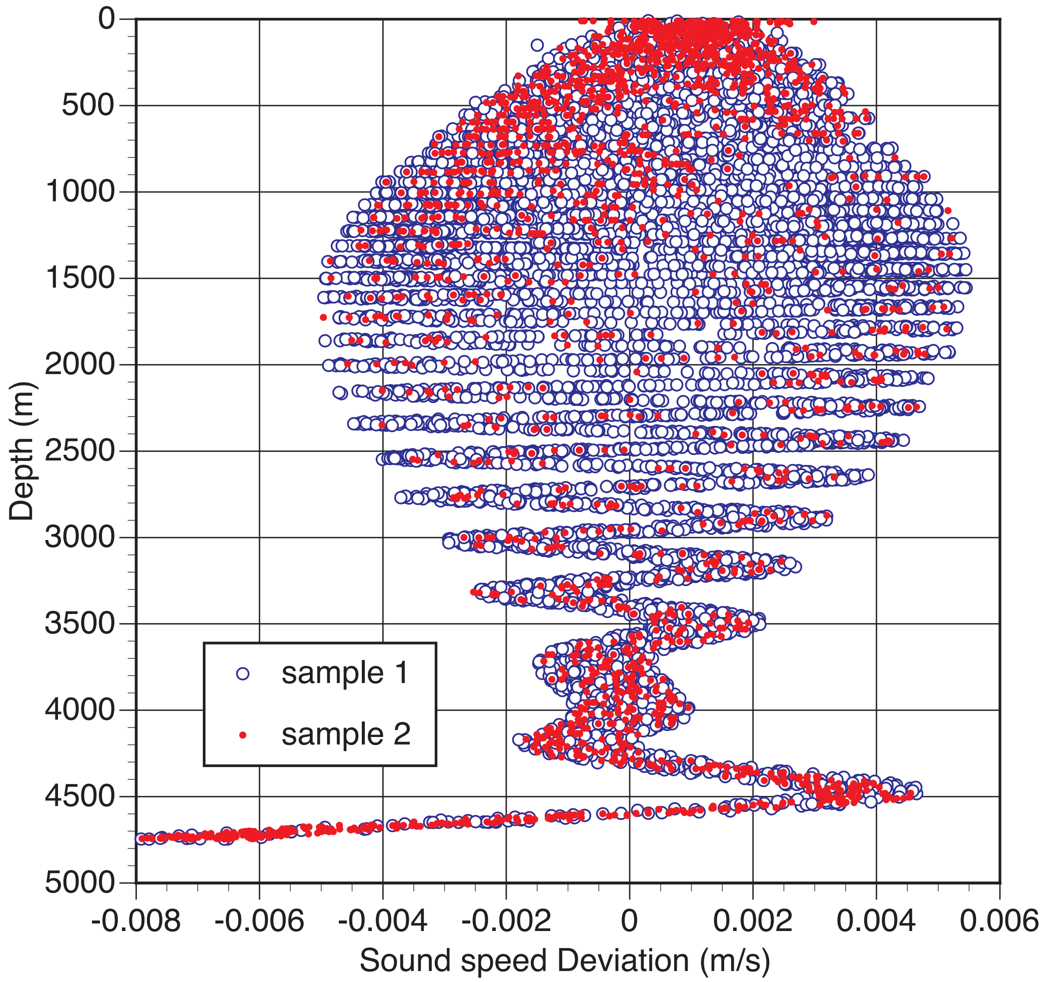

Figure 2. Plot of difference between analytic Munk Profile and the FeyRay interpolant using two different beam samplings.

3.2 Lloyds Mirror

The Lloyd's mirror test has been used primarily as a means of testing both the reflection from a totally reflecting surface, and the wide angle capabilities of parabolic approximation models. Since wide angle capability is not a problem for beam models, the Lloyd's mirror test was used to demonstrate the reflection capability and the interpolation components of the FeyRay and GRAB models. In theory a Gaussian beam model gains much of its computational advantage (over classic ray theory constructs) in its ability to properly model caustics. Gaussian beam models also support the ability to model large swatches of low environmental variability with a very few widely spaced beams. The Lloyd's mirror scenario used for this test is taken from the PE Workshop II specifications. The water depth was set to 5000 m, the source depth to 350 meters, and the receiver depth to 3990 meters. The frequency was 40 Hz. Since bottom returns are not a feature supported by this test, the bottom boundary condition for both models was set to a density ratio of 1.0, a sound speed ratio of 1.0, attenuation was set to 0.001 dB per wavelength to create a non-reflecting interface. Both models were run using default beam/ray fans. For GRAB this meant that the rays were specified from -89° to 89° in 0.10° increments. FeyRay internally selected its beams for this case. Figure 3 shows the benchmark solution, the GRAB solution and the FeyRay solution for ranges up to 10 km.

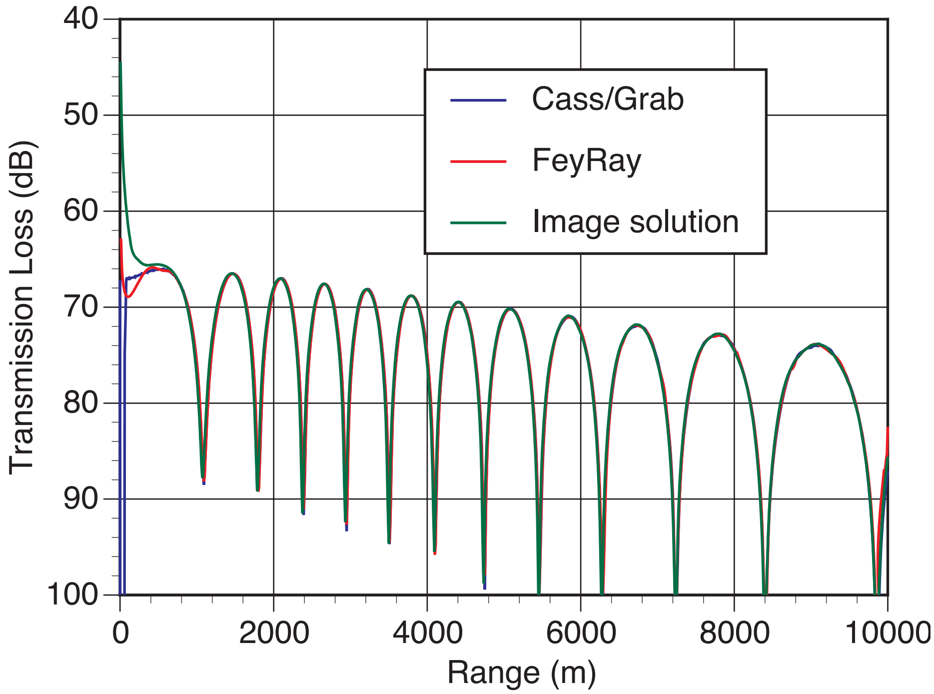

Figure 3 Plot of Lloyd's mirror transmission loss showing image solution, FeyRay solution, and GRAB solution.

The FeyRay and GRAB solutions agree with the benchmark solution for ranges beyond 600 meters. The difference at short ranges is best understood as an artifact of using the default launch angles. Better performance should be expected at short ranges for both models if launch angles were changed to improve the sampling of high angle paths.

3.3 Robustness Test

The robustness of the models was tested by computing transmission losses for one thousand twenty three distinct environments randomly selected from Norwegian, North Atlantic, Mediterranean, Western and Eastern Pacific areas. The seasons were selected such that spring, summer, fall, and winter were equiprobable. Surface loss was assumed to be zero. The model was exercised for four frequencies (50, 100, 250, and 2500 Hz) with one source depth, and three receiver depths for each test case. Each model was run with default beam/ray fan specifications (see the Lloyds mirror specification). NSPE solutions were used as the benchmark solution.

Each run was made as a stand alone process (as opposed to a library process) running in a batch file. Thus each run loaded a fresh executable in memory, and cleared memory upon completion. The robustness of the model was assessed by investigating whether each test case ran to normal completion with no errors issued by the program or operating system. The results are summarized in the set of figures below. The figures consist of histograms of distribution of the correlation error, and average error for each model relative to the benchmark for strong and medium targets.

Figure 4 shows the distribution of correlation error for FeyRay using a 100 dB threshold, and Figure 5 shows the distribution of correlation error for GRAB using a 100 dB threshold. The difference in the mean correlation error is small (0.06) and the difference in standard deviations is small (0.016) with FeyRay in demonstrating a smaller mean correlation error, and GRAB demonstrating a slightly tighter distribution. Figure 6 shows the distribution of average error for FeyRay using a 100 dB threshold, and Figure 7 shows the distribution of average error for GRAB using a 100 dB threshold. FeyRay has a smaller average error than GRAB (0.24 dB versus -4.79dB) with FeyRay predicting lower signal excess than the benchmark, and

GRAB predicting higher signal excess. GRAB distribution is more compact than the FeyRay distribution (2.79 dB versus 3.36 dB).

Figure 8 shows the distribution of correlation error for FeyRay using a 80 dB threshold, and Figure 9 shows the distribution of correlation error for GRAB using a 80 dB threshold. The difference in the mean correlation error is 0.109 and the difference in standard deviations is small (0.029) with FeyRay in demonstrating a smaller mean correlation error, and GRAB demonstrating a slightly tighter distribution. The difference in the correlation error, and standard deviation is larger for the 80 dB threshold although they the ratio of the numbers is approximately the same. Figure 10 shows the distribution of average error for FeyRay using a 80 dB threshold, and Figure 11 shows the distribution of average error for GRAB using a 80 dB threshold). FeyRay has a smaller average error than GRAB (-0.48 dB versus -1.76 dB) with both model predicting higher signal excess than the benchmark FeyRay distribution is more compact than the GRAB distribution (0.80 dB versus 1.54 dB).

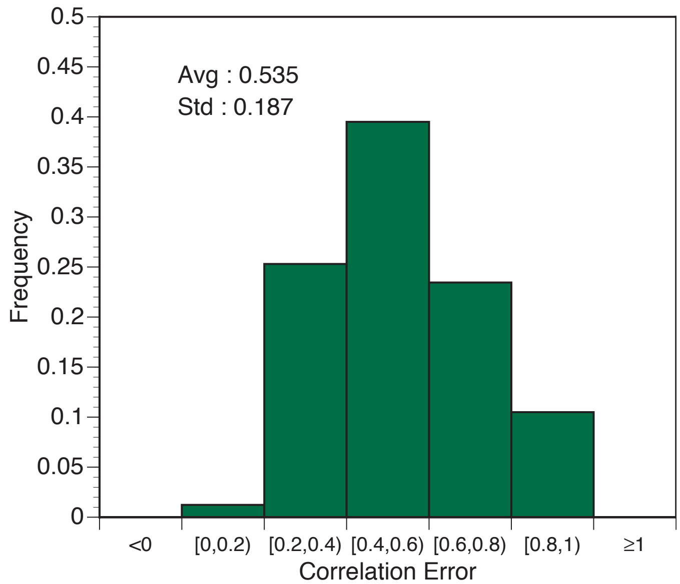

Figure 4 Histogram of correlation error for FeyRay robustness test for 100 dB threshold.

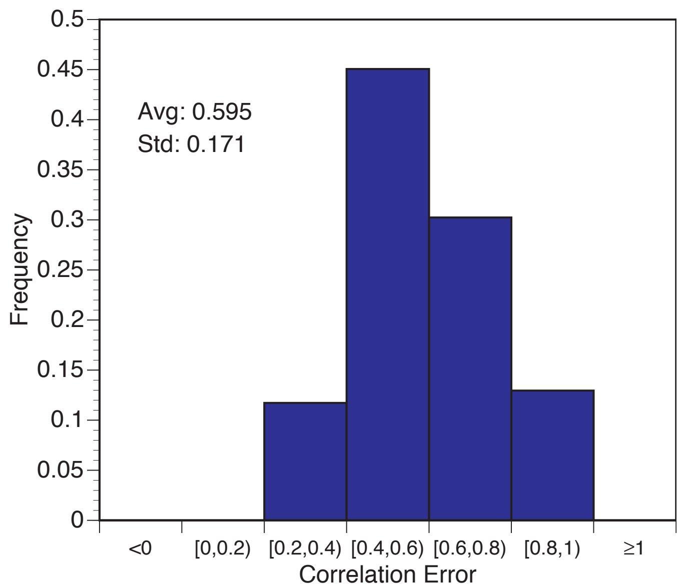

Figure 5 Histogram of correlation error for GRAB robustness for 100 db threshold.

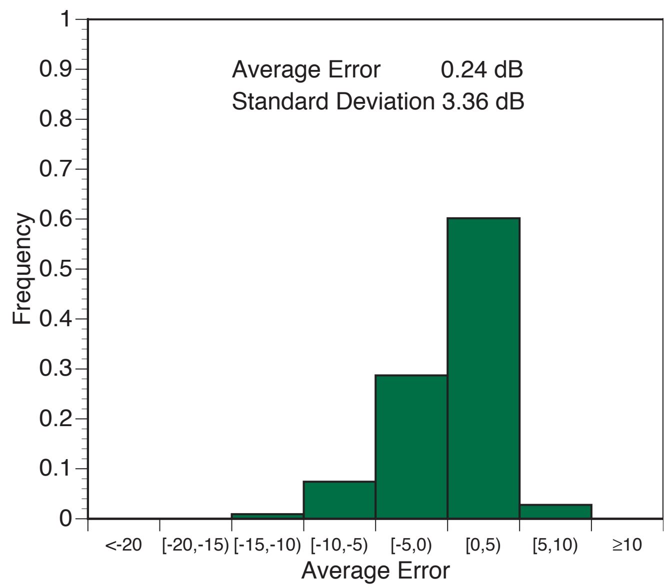

Figure 6 Histogram of average error for FeyRay Robustness test using 100 dB threshold.

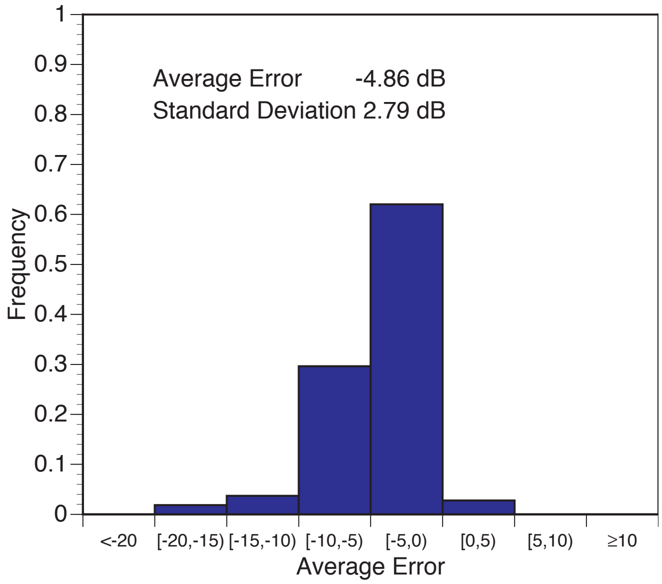

Figure 7 Histogram of average error for GRAB robustness test using 100 dB threshold.

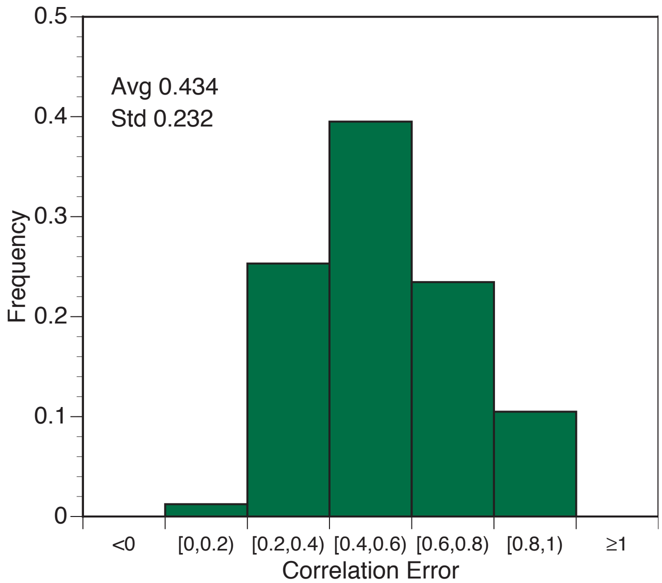

Figure 8 Histogram of correlation error for FeyRay robustness test using 80 dB threshold.

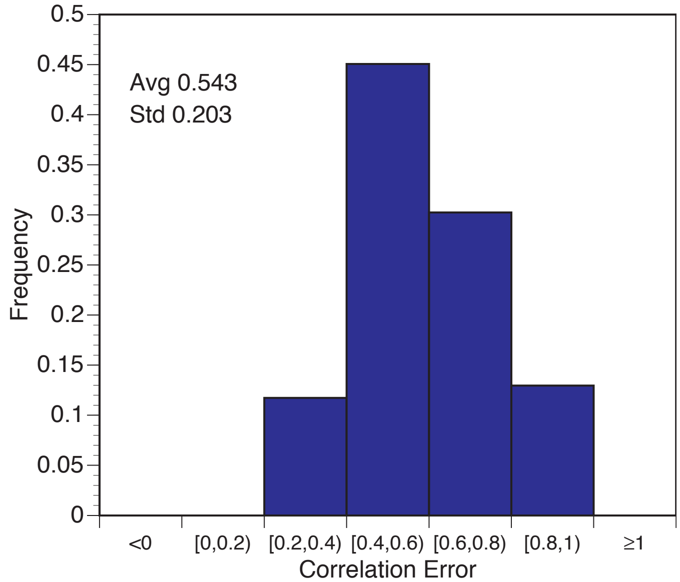

Figure 9 Histogram of correlation error for GRAB robustness test using 80 dB threshold.

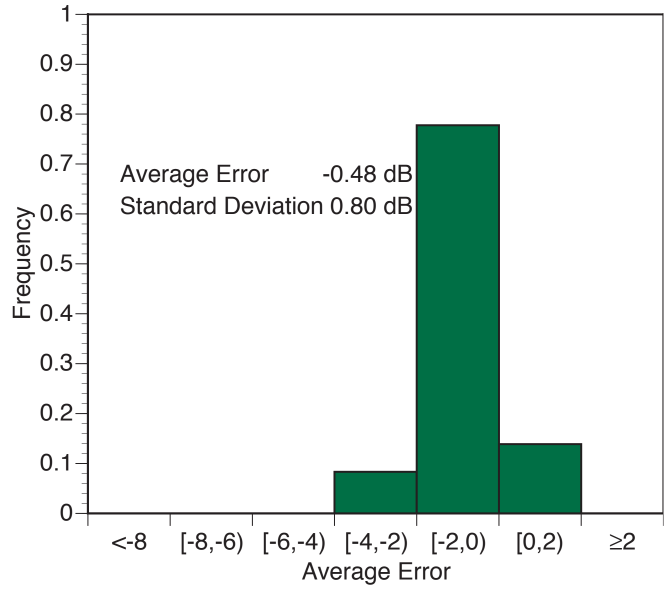

Figure 10 Histogram of average error for FeyRay robustness test using 80 dB threshold.

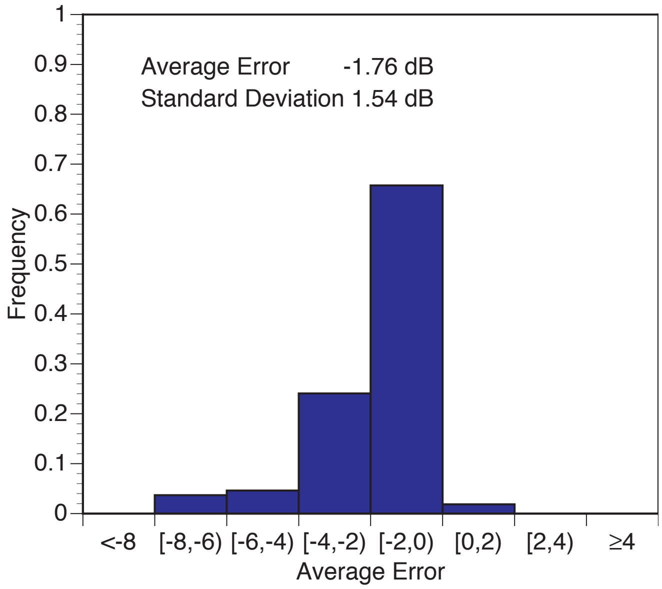

Figure 11 Histogram of average error for GRAB robustness test using 80 dB threshold.

3.4 Execution Speed

Since improved computational speed is one of the features touted by the FeyRay developer, timing comparisons were also performed on the GRAB and FeyRay models. Each of the models was modified slightly so that the time to compute transmission loss could be logged without including the time needed to load the program into memory. The time required to compute the transmission loss for each of the robustness tests was recorded and averaged. In each case, results and temporary files were written to a RAM disk to minimize the impact of disk I/O differences between the models. Note that these results are for the calculation of complete transmission loss curves from 0-140 km with output every 1/3 km.

GRAB comes in two flavors on the distribution disk: version 2 and version 3. GRAB version 2uses a triangular interpolation scheme for the sound velocity profile. GRAB version 3 uses a less accurate bi-linear interpolation that greatly improves the execution speed of GRAB. In the timing tests, both versions of GRAB were used. In the rest of this document, the more accurate version of GRAB was used.

These timing results were collected as part of the robustness tests described in Section 3.3. Each test case had 4 sound speed profiles, 140 km range, range dependent bathymetry (2500-5000 m), 2 bottom loss provinces and the default ray trace parameters Table 1 shows the performance. The run times reported here are for a specific computer, operating system, and compiler suite as well as the type of test case reported. Variations in performance ratios should be expected, as usage of cache, real and virtual memory are different for each of these models.

Table 1 Relative run time performance of FeyRay and GRAB.

Candidate Model	Average	Standard Deviation	Ratio (Model/FeyRay)
GRAB V2	145.204	1.384	20.82
GRAB V3	35.818	1.214	5.14
FeyRay	6.975	0.122	1.00

4.0 DEEP WATER ACCURACY TESTS

The deep water accuracy analysis focuses on areas where features in the sound speed profile are paramount. Three deep cases will now be considered. These cases are one using a range independent bottom with a range independent Munk profile (as illustrated in Figure 1), a range dependent case with a surface duct overlying a sound channel, and a range dependent case with a double duct capped by a surface duct.

4.1 Munk Profile

A simple test of model performance prediction is a convergence zone in deep water. The source is placed at 1000 m, the receiver at 800 m, the water depth is set to 5000 m. The bottom is defined as in the Lloyd's mirror case (density ratio 1.0, sound speed ratio 1.0, and attenuation as 0.001 dB per wavelength). The sound speed is depicted in Figure 1. The default ray fans for GRAB, and beam fans for FeyRay are selected. Propagation range is set to 100 km.

Figure 12 shows the prediction of the PE model, FeyRay, and GRAB as functions of range for a frequency of 25 Hz. Both GRAB and FeyRay track the PE solution through the first CZ when GRAB shows a tendency to under-predict the transmission loss. Figure 13 shows the transmission loss prediction of the PE model, FeyRay, and GRAB as functions of range for a frequency of 100 Hz. The structure of the predictions are similar, except that FeyRay suffers a large over-prediction of loss between the ranges of 20 and 35 km. Figure 14 shows the prediction of the PE model, FeyRay and GRAB as functions of range for a frequency of 250 Hz. At 250 Hz FeyRay exhibits the same over-prediction of transmission loss between the ranges of 20 and 35 km as in the 100 Hz case. GRAB is tracking the null beyond the first CZ properly. Figure 15 shows the predictions of the PE model, FeyRay, and GRAB as functions of range for a frequency of 2500 Hz. GRAB and FeyRay continue to produce similar outputs to the 250 Hz case, with the same structure.

Table 2 shows the correlation error , average error, and maximum detection range in km for FeyRay and GRAB for a 60 dB threshold. This table includes the maximum detection range for PE. The ranges are short, thus the number of data points that go into the error and correlation error are small. Both FeyRay, and GRAB predict the PE solution well.

Table 2 Correlation error, average error, and maximum detection range for FeyRay, and GRAB for the Munk profile range independent case for 60 dB threshold.

Freq.	FeyRay corr. Err	FeyRay error	GRAB corr err	GRAB error	Max range PE	Max range FeyRay	Max range GRAB
25	0 %	0.0 dB	0 %	0.0 dB	1 km	1 km	1 km
100	4 %	0.2 dB	19 %	0.4 dB	1 km	2 km	1 km
250	0 %	0.0 dB	1 %	0.2 dB	2 km	2 km	2 km
2500	3 %	0.2 dB	2 %	0.2 dB	2 km	1 km	1 km

Table 3 shows the correlation error and average error for FeyRay and GRAB for an 80 dB threshold. The correlation error for both models is small, and the average error is under 0.5 dB for all frequencies.

Table 3 Correlation error and average error for FeyRay and GRAB for the Munk profile range independent case or 80 dB threshold..

Frequency	FeyRay Correlation Error	FeyRay error	GRAB Correlation Error	GRAB error
25	1 %	0.1 dB	2 %	0.0 dB
100	10 %	0.1 dB	11 %	0.0 dB
250	12 %	0.0 dB	15 %	0.0 dB
2500	9 %	0.4 dB	17 %	0.2 dB

Table 4 shows the minimum and maximum range of detection for PE, FeyRay, and GRAB for an 80 dB threshold. FeyRay and GRAB show minimum and maximum ranges consistent with PE, except for the 2500 Hz case which appears anomalous.

Table 4 Minimum and maximum range of detection ranges (in km) for PE, FeyRay and GRAB for an 80 dB threshold for the Munk profile case (maximum range is 100 km)

Frequency	PE min	FeyRay min	GRAB min	PE max	FeyRay max	GRAB max
25	2.5 km	2.5 km	2.5 km	56.5 km	12.5 km	61 km
100	8.5 km	8.5 km	6.5 km	59.5 km	54 km	55.5 km
250	3 km	10 km	8.5 km	58 km	40.5 km	56.5 km
2500	8.5 km	3 km	3 km	89 km	12 km	12 km

Table 5 shows the correlation error for FeyRay and GRAB for an 100 dB threshold. The correlation error for both models is small, and the average error is under 2.0 dB for all frequencies except the possibly anomalous 2500 Hz case.

Table 5 Correlation error for FeyRay and GRAB for 100 dB threshold case for the Munk profile..

Frequency	FeyRay Correlation Error	FeyRay error	GRAB coefficient of error	GRAB error
25	7 %	1.0 dB	10 %	-0.7 dB
100	10 %	1.1 dB	13 %	0.1 dB
250	12 %	1.7 dB	15 %	-0.7 dB
2500	18 %	9.0 dB	25 %	7.7 dB

Table 6 shows the minimum and maximum range of detection for PE, FeyRay, and GRAB for an 100 dB threshold. FeyRay and GRAB show minimum and maximum ranges consistent with PE

Table 6 Minimum and maximum detection ranges (in km) for PE, FeyRay, and GRAB for an 100 dB threshold for the Munk profile case (maximum range is 100 km).

Frequency	PE min	FeyRay min	GRAB min	PE max	FeyRay max	GRAB max
25	14.5 km	14.5 km	14.5 km	all	all	all
100	13 km	13.5 km	15 km	all	all	all
250	12.5 km	15 km	15 km	all	all	all
2500	14.5 km	15 km	12.5 km	all	90 km	99 km

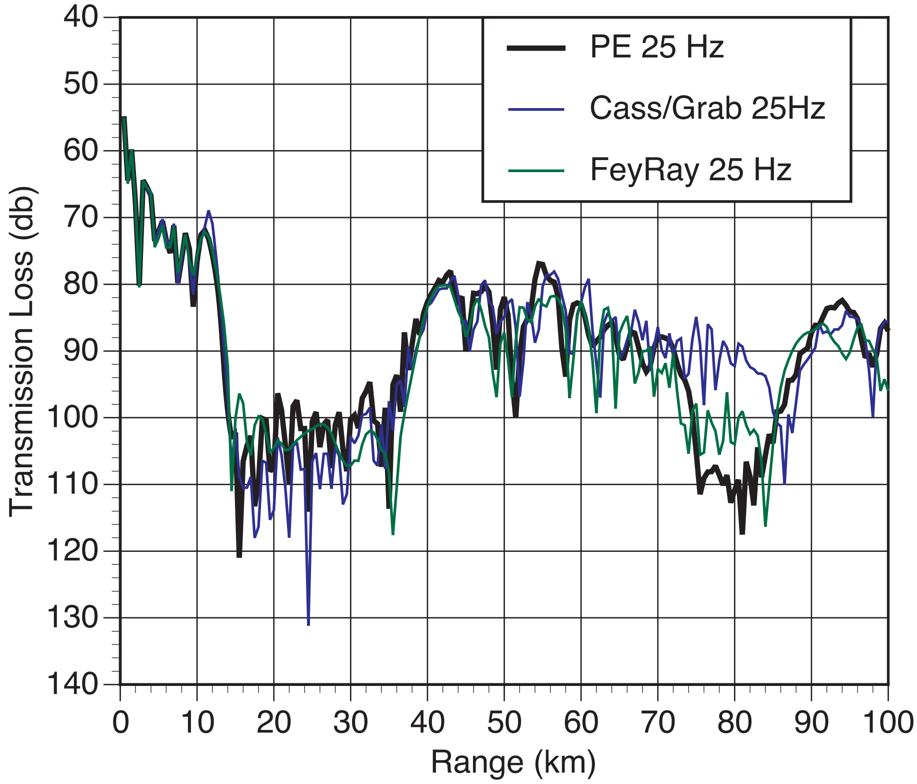

Figure 12 Plot of transmission loss for PE, FeyRay, and GRAB for the range independent Munk profile for a frequency of 25 Hz.

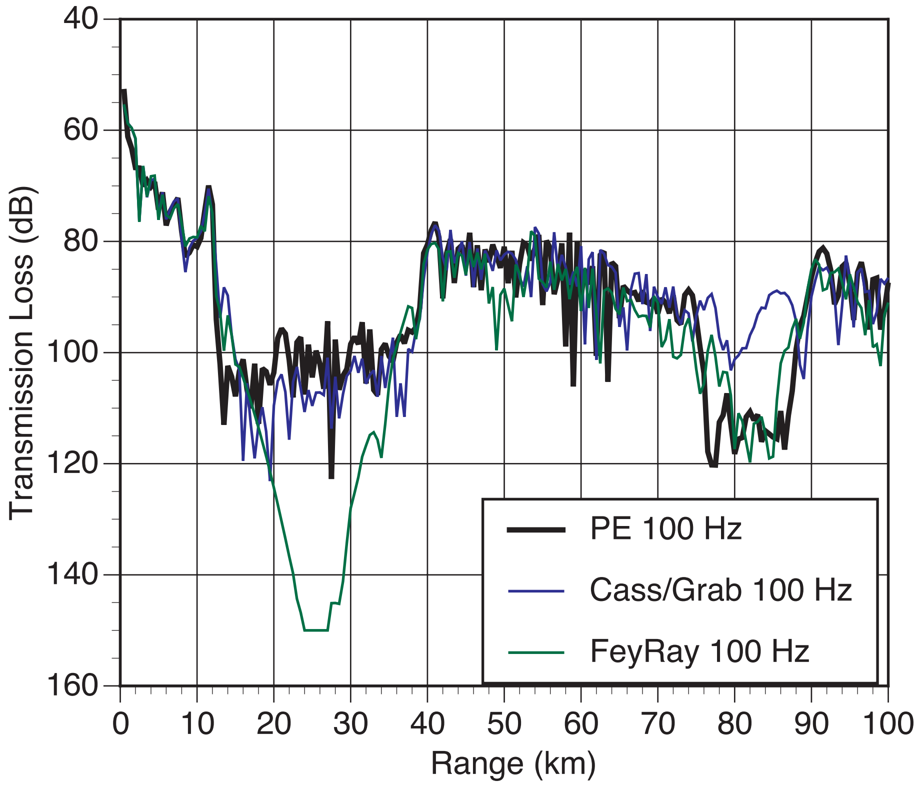

Figure 13 Plot of transmission loss for PE, FeyRay, and GRAB for the range independent Munk profile for a frequency of 100 Hz.

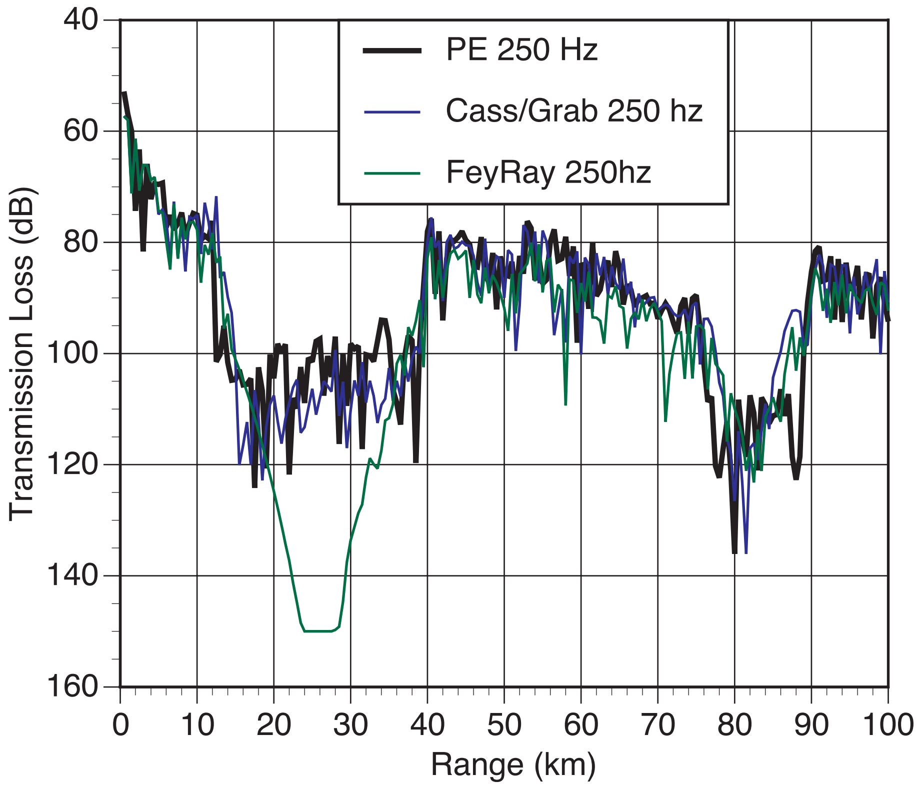

Figure 14 Plot of transmission loss for PE, FeyRay and GRAB for the range independent Munk profile case for a frequency of 250 Hz.

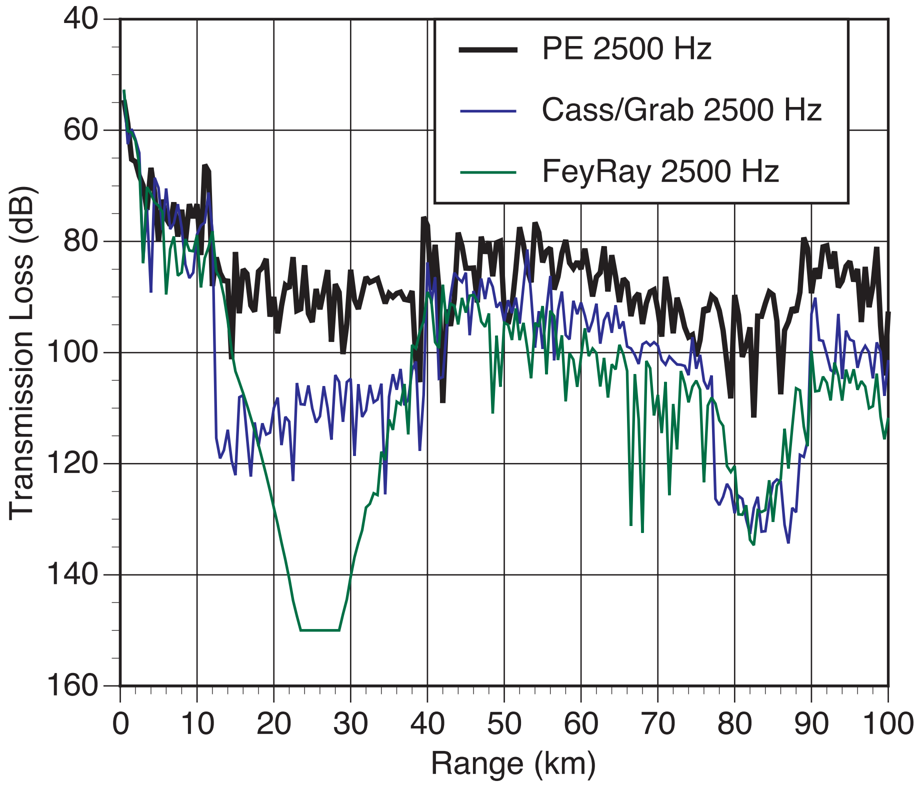

Figure 15 Plot of transmission loss for PE, FeyRay and GRAB for the range independent Munk profile case for a frequency of 2500 Hz.

4.2 Surface Duct over Deep Duct

The surface duct over deep duct test case (also referred to as wlan069) features a source depth of 25.4 meters and a receiver depth of 385.1 meters. Figure 16 depicts the environment. The bottom for this case is made a constant silty-clay (density 1.488, sound speed 1.014, attenuation 0.1 dB per wavelength). Analysis of the environment shows that the source is located within the surface duct, and the receiver below the base of the surface duct. There is some variability in the location and strength of the surface duct as a function of range, thus a tensor spline approach to fitting the sound speed field may lead to problems. FeyRay and GRAB are both exercised using their default ray/beam fan definitions.

The prediction for PE model , FeyRay, and GRAB for 25 Hz are show in Figure 17. At 25 Hz GRAB has a tendency to under-predict the transmission losses for this frequency. Figure 18 shows the predictions for PE model, FeyRay, and GRAB at 100 Hz. At this frequency the GRAB model more clearly follows the level of the PE model, while FeyRay is over-predicting the transmission loss at range. Figure 19 shows the PE model, FeyRay, and GRAB predictions for 250 Hz at a function of range. At 250 Hz both FeyRay, and GRAB predictions follow the PE solution, except for a minor excursion in the FeyRay solution near 90 km range. Figure 20 shows the PE model, FeyRay, and GRAB predictions for 250 Hz at a function of range The 2500 Hz solution for PE model , FeyRay, and GRAB has FeyRay and GRAB agreeing at range against PE. In this scenario, results will not be shown for the 60 dB threshold because the environment is such that the weak target is rarely detected.

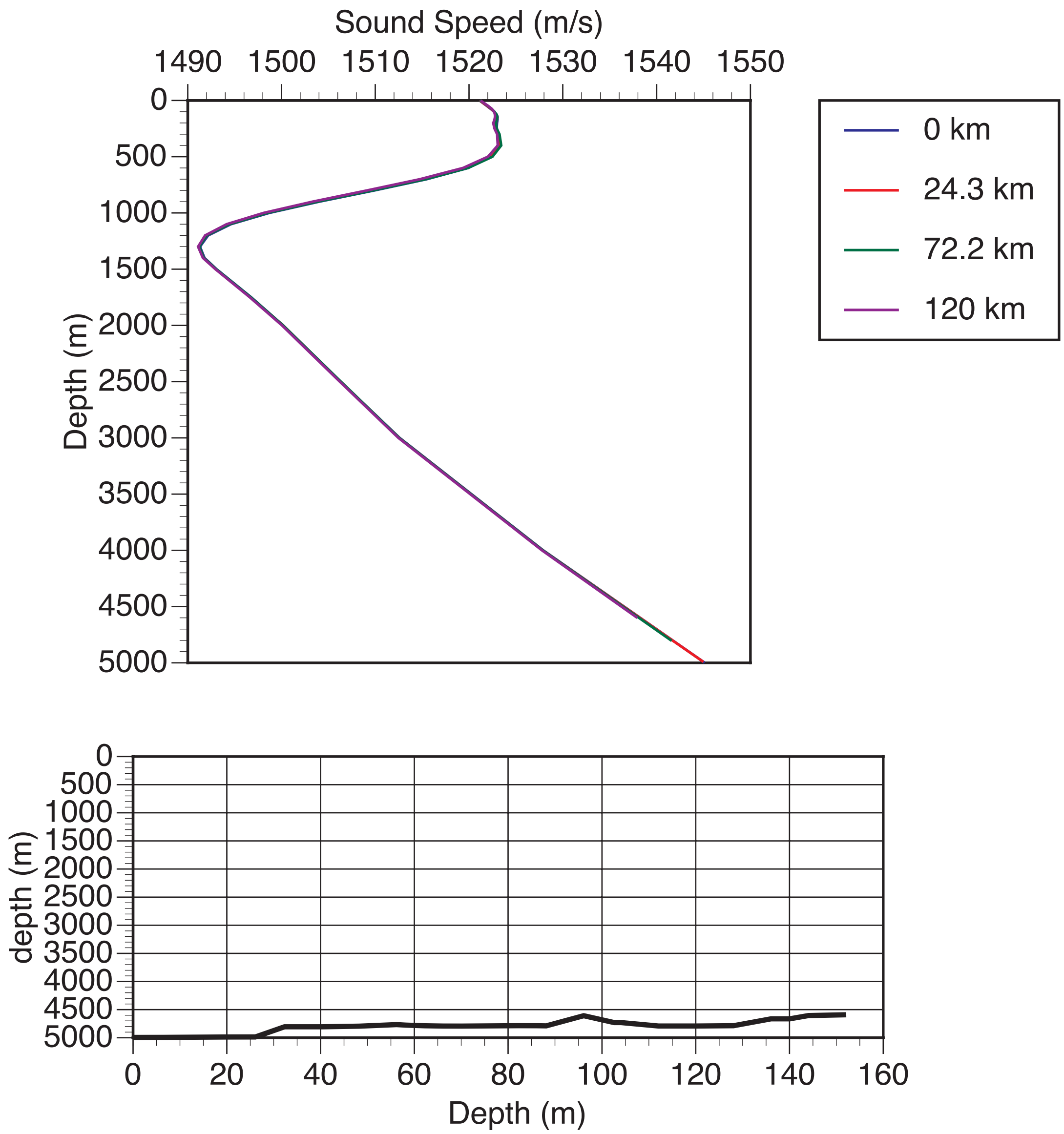

Figure 16 Sound speed and bathymetric environment for surface duct over deep duct test case (wlan 069).

Table 7 shows the correlation error for FeyRay and GRAB for an 80 dB threshold. The correlation error for both models is small, and the average error is under 0.3 dB for all frequencies. The overall appearance seen in the transmission loss plots translates into excellent statistical fits.

Table 7 Correlation error and average error for FeyRay and GRAB for the surface duct over deep channel profile range dependent case (wlan069 extraction) for 80 dB threshold.

Frequency	FeyRay Correlation Error	FeyRay error	GRAB coefficient of error	GRAB error
25	2 %	-0.1 dB	7 %	-0.2 dB
100	3 %	0.1 dB	27 %	-0.3 dB
250	7 %	0.2 dB	27 %	0.0 dB
2500	20 %	0.2 dB	21 %	0.1 dB

Table 8 shows the minimum and maximum range of detection for PE, FeyRay, and GRAB for an 80 dB threshold. FeyRay and GRAB show minimum and maximum ranges consistent with PE, except for the 2500 Hz case which appears anomalous.

Table 8 Minimum and maximum ranges (in km) of detection for PE, FeyRay, and GRAB for the surface duct over deep channel range dependent profile case (wlan069 extraction) for 80 dB threshold (maximum range is 140 km).

Frequency	PE min	FeyRay min	GRAB min	PE max	FeyRay max	GRAB max
25	5 km	5.5 km	3 km	5.5 km	13 km	34 km
100	11.5 km	7 km	11 km	68 km	62 km	89 km
250	3.5 km	3.5 km	7.5 km	67 km	68 km	68 km
2500	6.5 km	4.5 km	4.5 km	64 km	8.5 km	21 km

Table 9 shows the correlation error for FeyRay and GRAB for an 100 dB threshold. The correlation error is still small for both GRAB and FeyRay with the possible exception of the 250 Hz GRAB run. GRAB performs excellently as measured by transmission loss except at 25 Hz, and FeyRay performs excellently except at 2500 Hz.

Table 9 Correlation error for FeyRay and GRAB for the surface duct over deep channel profile range dependent case (wlan069 extraction) for 100 dB threshold.

Frequency	FeyRay Correlation Error	FeyRay error	GRAB coefficient of error	GRAB error
25	25 %	-1.8 dB	37 %	-6.3 dB
100	39 %	3.1 dB	39 %	-2.7 dB
250	37 %	3.2 dB	51 %	-1.4 dB
2500	28 %	7.3 dB	34 %	3.8 dB

Table 10 shows the minimum and maximum range of detection for PE, FeyRay, and GRAB for an 80 dB threshold. FeyRay and GRAB show minimum and maximum ranges consistent with PE

Table 10 Minimum and maximum range (in km) of signal excess for PE, FeyRay and GRAB for the surface duct over deep channel profile range dependent case (wlan069 extraction) for 100 dB threshold (maximum range is 140 km).

Frequency	PE min	FeyRay min	GRAB min	PE max	FeyRay max	GRAB max
25	12 km	15.5 km	26 km	139 km	all	all
100	33.5 km	12.5 km	37.5 km	all	all	all
250	32 km	26 km	12.5 km	all	all	all
2500	28.5 km	21.5 km	13 km	all	84.5 km	125.5 km

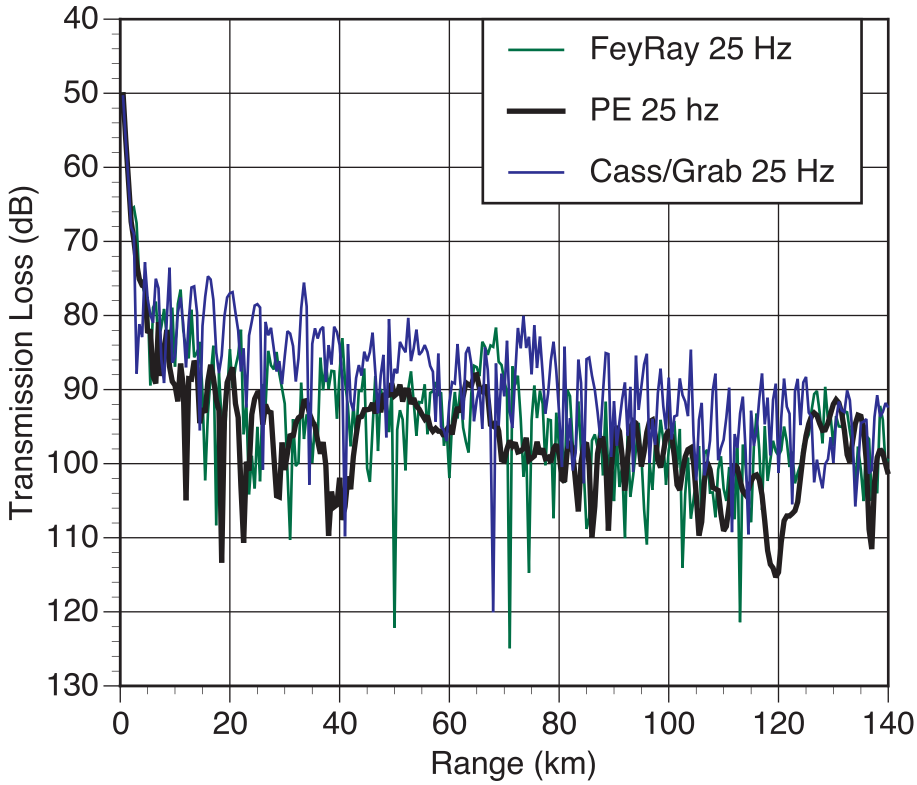

Figure 17 Plot of predicted transmission loss for PE, FeyRay, and GRAB for surface duct over deep channel profile range dependent case (wlan069 extraction) for a frequency of 25 Hz.

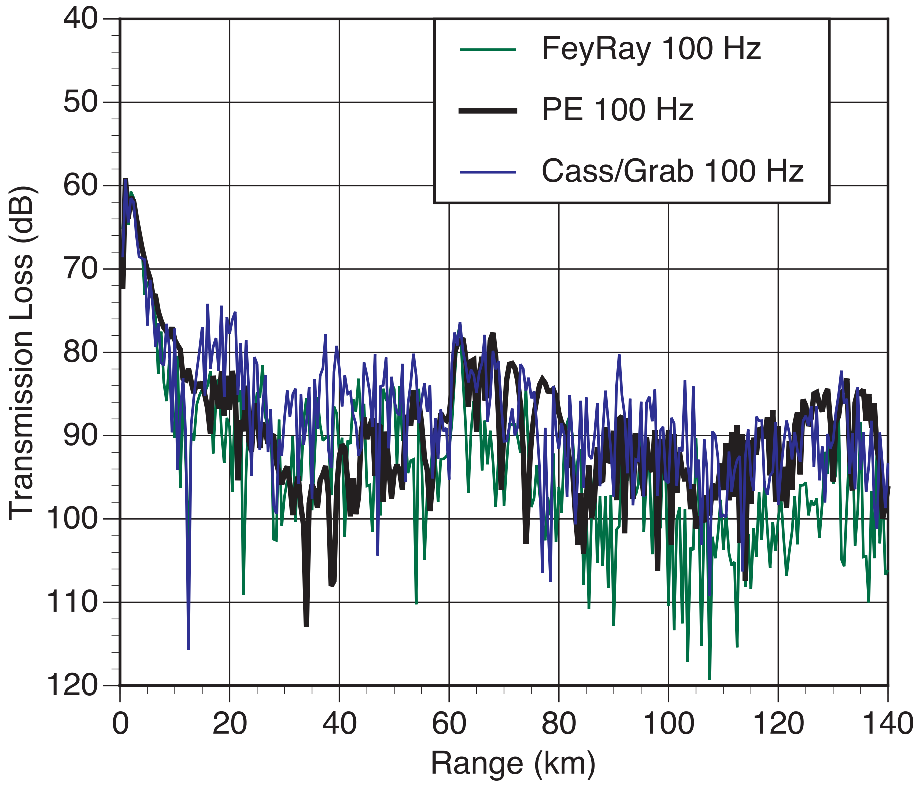

Figure 18 Plot of predicted transmission loss for PE, FeyRay, and GRAB for surface duct over deep channel profile range dependent case (wlan069 extraction) for a frequency of 100 Hz.

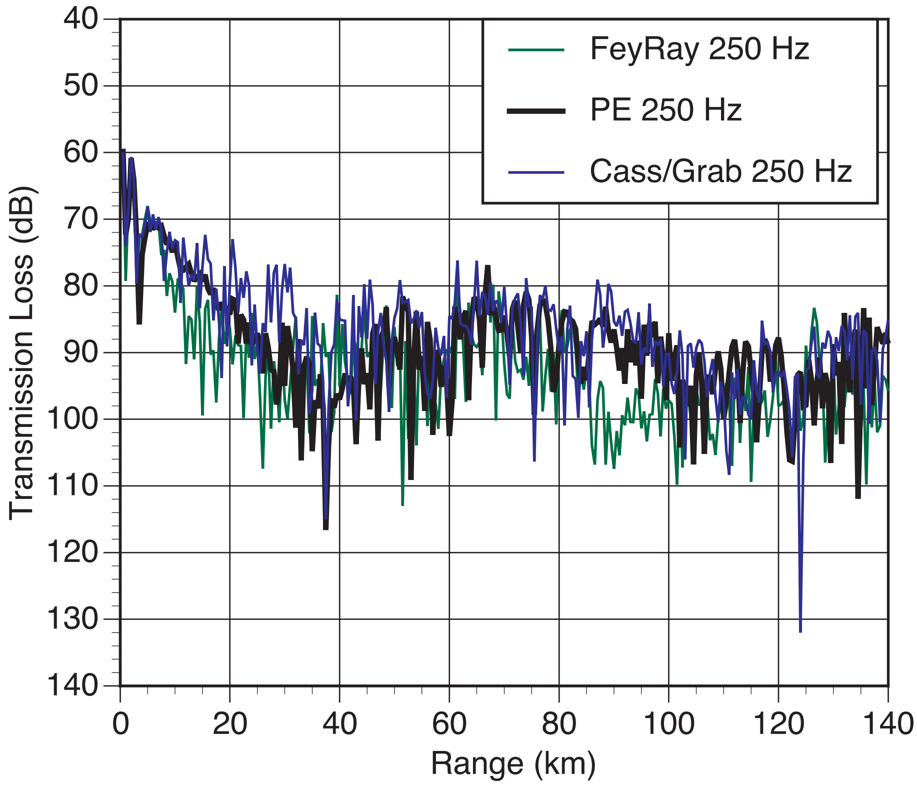

Figure 19 Plot of predicted transmission loss for PE, FeyRay, and GRAB for surface duct over deep channel profile range dependent case (wlan069 extraction) for a frequency of 250 Hz.

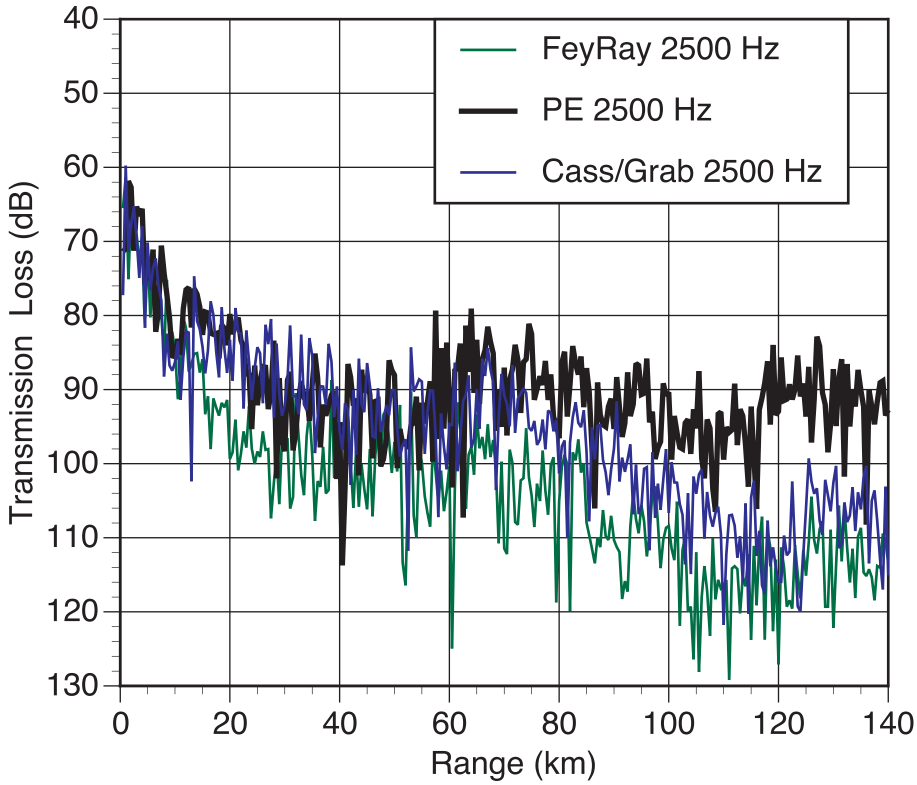

Figure 20 Plot of predicted transmission loss for PE, FeyRay, and GRAB for surface duct over deep channel profile range dependent case (wlan069 extraction) for a frequency of 2500 Hz.

4.3 *Surface Duct over Double Duct*

The surface duct over double duct test case (also referred to as wlan183) features a source depth of 25.4 meters and a receiver depth of 385.1 meters. Figure 21 depicts the environment. The bottom for this case is made a constant silty-clay (density 1.488, sound speed 1.014, attenuation 0.1 dB per wavelength). Analysis of the environment shows that the source is located within the surface duct, and the receiver near the channel axis of the first deep duct. There is some variability in the location and strength of the surface duct as a function of range, and some variability in the depths, and strengths of the deep ducts. Again the tensor spline approach to fitting the sound speed field may lead to problems. FeyRay and GRAB are both exercised using their default ray/beam fan definitions. In this scenario, results will not be shown for the 60 dB threshold because the environment is such that the weak target is rarely detected.

Figure 22 shows the solutions generated by PE, FeyRay, and GRAB for this test case at a frequency of 25 Hz. The solutions show FeyRay tracking the PE solution quite well, while GRAB is predicting too little loss with very little structure. Figure 23 shows the solutions generated by PE, FeyRay and GRAB for this test case at 100 Hz. At this frequency GRAB solution is closer in appearance to PE solution than the previous case. The FeyRay solution tends to track the PE solution by the levels are too low. The 250 Hz solutions for PE, FeyRay and GRAB are shown in Figure 24. The trends observed at 25 Hz, and 100 Hz continue. There is a tendency for GRAB solutions to be optimistic at the mid-point between profiles, and the FeyRay solutions to be pessimistic. The 2500 Hz case shown in Figure 25 for PE, FeyRay and GRAB shows the continuation of the previous trends in FeyRay. That is at the mid-point between sound speed fields a minimum whose effects increases with frequency occurs. The GRAB tendency to underestimate loss disappears.

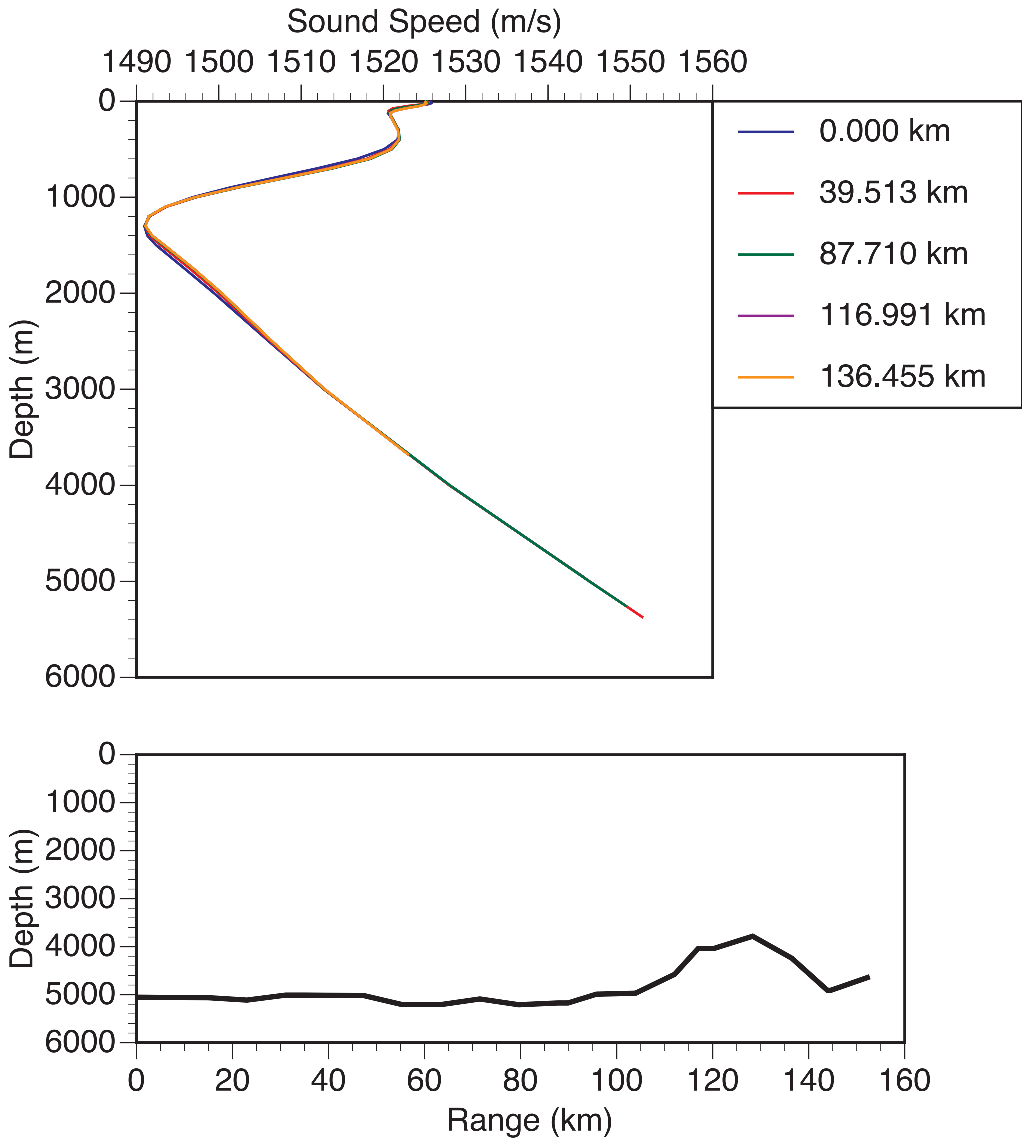

Figure 21 Sound speed and bathymetric environment for surface duct over double duct range dependent case (wlan183 extraction).

Table 11 shows the correlation error for FeyRay and GRAB for an 80 dB threshold. The correlation error for both models is small, and the average error is under 0.2 dB for all frequencies. It should be noted that the sample for this estimate is small.

Table 11 Correlation error and average error for FeyRay, and GRAB for the surface duct over double duct profile range dependent environment (wlan183 extraction) for 80 dB threshold.

Frequency	FeyRay Correlation Error	FeyRay error	GRAB coefficient of error	GRAB error
25	3 %	-0.2 dB	12 %	-0.2 dB
100	23 %	0.0 dB	26 %	-0.2 dB
250	16 %	-0.1 dB	32 %	-0.2 dB
2500	12 %	0.2 dB	12 %	0.0 dB

Table 12 shows the minimum and maximum range of detection for PE, FeyRay, and GRAB for an 80 dB threshold. FeyRay and GRAB show minimum and maximum ranges consistent with PE, except for the FeyRay at 100 Hz, and GRAB at 250 Hz and the 2500 Hz case which appears anomalous.

Table 12 Minimum and maximum detection ranges (in km) for PE, FeyRay, and GRAB for the surface duct over double duct profile range dependent environment (wlan183 extraction) for 80 dB threshold (maximum detection range is 140 km).

Frequency	PE min	FeyRay min	GRAB min	PE max	FeyRay max	GRAB max
25	4 km	4 km	3.5 km	5 km	13 km	54.5 km
100	0.5 km	5.5 km	1.5 km	126 km	9 km	64 km
250	6.5 km	5 km	7 km	63.5 km	64 km	125 km
2500	7 km	5.5 km	6.5 km	63 km	8 km	8.5 km

Table 13 shows the correlation error for FeyRay and GRAB for an 100 dB threshold. In general FeyRay has a better correlation error than does GRAB. FeyRay outperforms GRAB at 25 Hz, GRAB outperforms FeyRay at 2500 Hz. Between those two frequencies there is a slight edge for FeyRay.

Table 13 Correlation error and average error for FeyRay, and GRAB for the surface duct over double duct profile range dependent environment (wlan183 extraction) for 100 dB.

Frequency	FeyRay Correlation Error	FeyRay error	GRAB coefficient of error	GRAB error
25	20 %	-0.5 dB	33 %	-7.7 dB
100	34 %	3.0 dB	51 %	-3.5 dB
250	35 %	3.3 dB	55 %	-4.6 dB
2500	33 %	7.3 dB	40 %	2.7 dB

Table 14 shows the minimum and maximum range of detection for PE, FeyRay, and GRAB for a 100 dB threshold. FeyRay and GRAB show minimum and maximum ranges consistent with PE

Table 14 Minimum and maximum detection ranges (in km) for PE, FeyRay, and GRAB for the surface duct over double duct profile range dependent environment (wlan183 extraction) for 100 dB threshold (maximum detection range is 140 km).

Frequency	PE min	FeyRay min	GRAB min	PE max	FeyRay max	GRAB max
25	12 km	15.0 km	29.5 km	134 km	all	all
100	12 km	22.5 km	22.5 km	all	139.5 km	all
250	11 km	15.5 km	13.5 km	all	all	all
2500	14 km	8 km	20 km	all	75 km	130 km

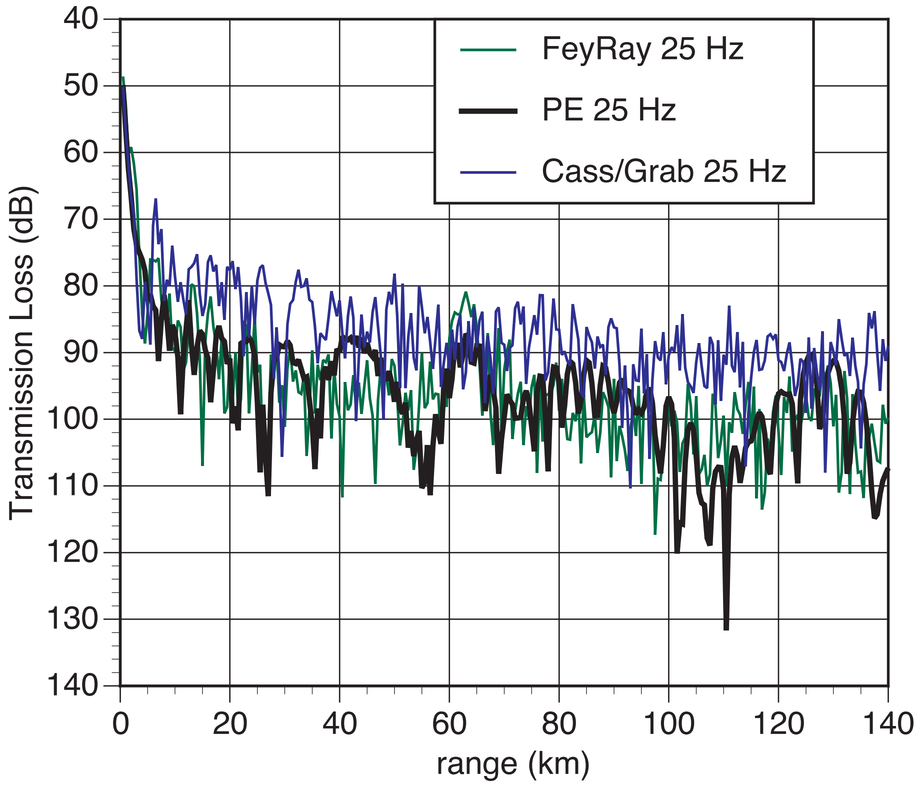

Figure 22 Plot of transmission loss predictions for PE, FeyRay, and GRAB for surface duct over double duct range dependent environment (wlan183 extraction) for a frequency of 25 Hz.

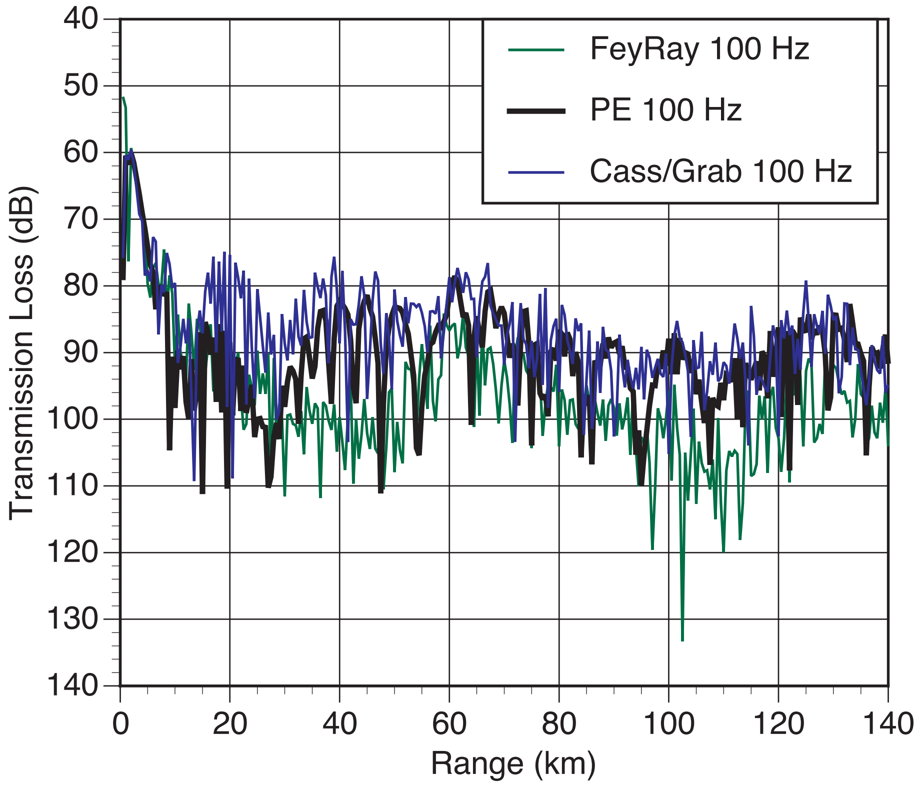

Figure 23 Plot of transmission loss predictions for PE, FeyRay, and GRAB for surface duct over double duct profile range dependent environment (wlan183 extraction) for a frequency of 100 Hz.

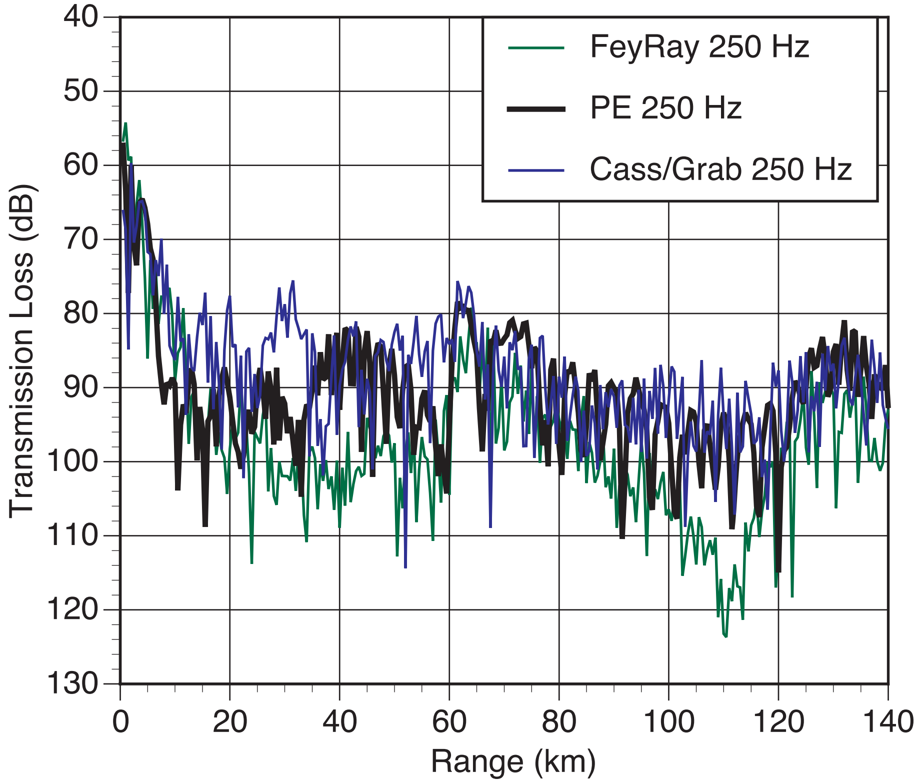

Figure 24 Plot of transmission loss predictions for PE, FeyRay, and GRAB for surface duct over double duct profile range dependent environment (wlan183 extraction) for a frequency of 250 Hz.

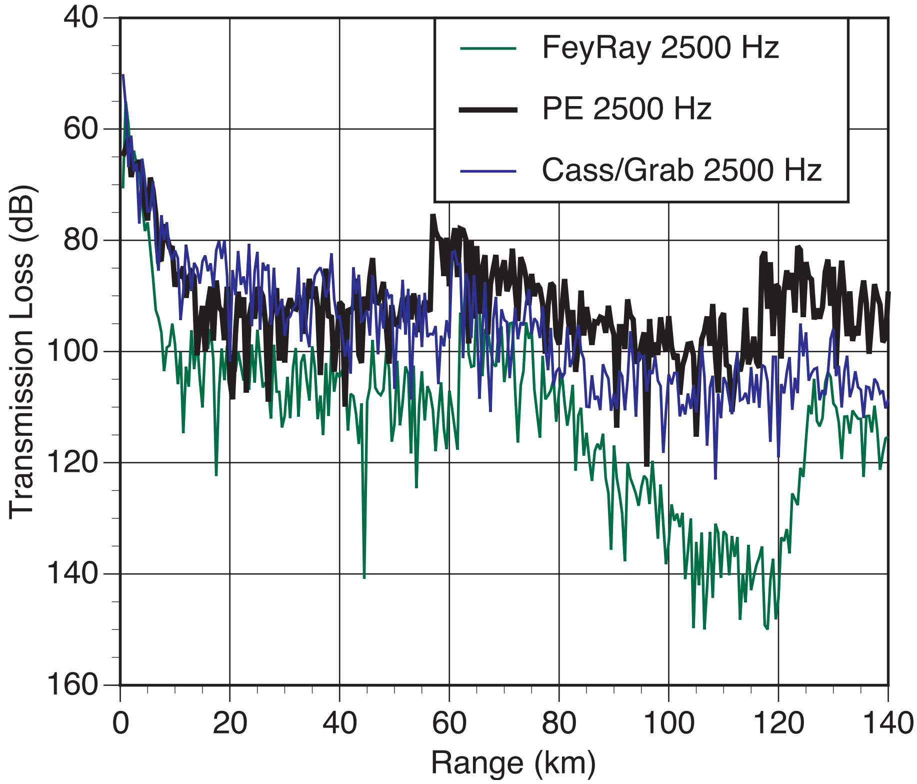

Figure 25 Plot of transmission loss predictions for PE, FeyRay, and GRAB for surface duct over double duct profile range dependent environment (wlan183 extraction) for a frequency of 2500 Hz.

4.4 *Summary of Deep Water Cases*

Three deep water cases were presented, the range independent Munk profile case, the surface duct over a deep channel case, and the surface duct over a double duct case. Two rather simple observations can be made: in general FeyRay had a lower correlation of error than did GRAB relative to a given threshold, and FeyRay performed better at lower frequencies that did GRAB. Both FeyRay and GRAB generated solutions that exhibited structural problems: that is they either created a sound channel where none existed, or ignored one which existed.

5.0 SHALLOW WATER ACCURACY TESTS

In this section the performance of FeyRay and GRAB will shown on three specific shallow water test cases. The three cases are the ASA benchmark wedge, a range independent environment with an upwardly refracting sound speed, and sea-test results from impulsive source data set in the East China Sea. Each of the first two data sets will be presented for 25, 100, 250, and 2500 Hz. The 25 Hz case was represents the model's ability to stretch into the low frequency domain; please note that GRAB's OAML certification is only down to 150 Hz. High frequency cases were limited to 2500 Hz to prevent excessive run times by NSPE (which already required all weekend to execute). The East China Sea data set will be presented for 25, 50, 100, 200, 400, and 800 Hz to match the conditions of the sea-test.

5.1 *ASA Wedge*

Although not representative of any real sediment the parameters of the Acoustical Society of America's (ASA) benchmark wedge predicts bottom loss behavior that is representative of the low loss in a sediment covered area. Use of a constant sound speed in the water column, as opposed to a downwardly refracting sound speed, leads to a harder propagation case since beams must be traced over longer ranges to obtain a reasonable answer. The constant sound speed in the water column is maintained to avoid interpolation differences between the two propagation models. A shallow water ASA benchmark wedge bottom loss case is then defined by:

- constant sound speed (1500 m/s),
- constant water depth (200 m) ,
- source depth (100 m), receiver depth (50 m),
- sediment sound speed (1700 m/s),
- sediment density (1.5 time water density), and
- sediment attenuation (0.5 dB per wavelength).

Transmission loss predictions are examined at 25, 100, 250, and 2500 Hz for NSPE, GRAB and FeyRay.

Figure 26 shows the FeyRay, GRAB and NSPE prediction at 25 Hz. In general neither FeyRay or GRAB match the benchmark solution, misplaced nulls are common for both models. Figure 27 shows the FeyRay, GRAB and PE prediction at 100 Hz. Both FeyRay and GRAB show better visual agreement with the benchmark solution, although variations are evident. Figure 28 shows the FeyRay, GRAB and PE predictions at 250 Hz. As expected, at higher frequencies the general level of the FeyRay and GRAB are much close to the benchmark solution. Again, as in the case of 100 Hz, there are clearly misalignments of peaks and troughs. Figure 29 shows the FeyRay, GRAB, and PE predictions at 2500 Hz. In this case the FeyRay, and GRAB solutions track the PE solutions quite well. The FeyRay solution shows more negative bias in transmission loss than does GRAB, with more obvious interference nulls.

Table 15 shows the FeyRay and GRAB correlation error, and average error for the ASA wedge case for a 60 dB threshold. Also shown is the maximum detection range predicted for each model. The maximum detection range is the range at which the predicted signal level exceeds the threshold level.

Table 15. Correlation error, average error and maximum detection range for FeyRay and GRAB for the ASA wedge for a 60 dB threshold (maximum detection range is 3.35 km).

Freq.	FeyRay corr. err	FeyRay error	GRAB corr err	GRAB error	Max range PE	Max range FeyRay	Max range GRAB
25	40 %	5.1 dB	39 %	3.2 dB	all	2.4 km	all
100	26 %	4.5 dB	20 %	3.6 dB	all	3.2 km	all
250	31 %	4.7 dB	37 %	4.9 dB	all	all	all
2500	34 %	4.8 dB	37 %	4.8 dB	all	all	all

Table 16 shows the summary of the correlation error , and average error for FeyRay and GRAB for a 80 dB threshold. Two observations can be made, the average error in GRAB is lower than the average error in FeyRay, and the correlation error is lower in FeyRay than in GRAB. GRAB produces a prediction that is closer to the benchmark, but the shape of the prediction is better in FeyRay.

Table 16 Correlation error and average error for FeyRay and GRAB for an 80 dB threshold for the ASA wedge problem.

Frequency	FeyRay corr error	FeyRay average error	GRAB corr error	GRAB average error
25	40 %	8.7 dB	38 %	3.8 dB
100	44 %	4.2 dB	35 %	1.2 dB
250	45 %	3.4 dB	51 %	1.2 dB
2500	46 %	2.9 dB	51 %	1.1 dB

Table 17 shows the minimum and maximum range of detection for PE, FeyRay, and GRAB for an 80 dB threshold. FeyRay and GRAB show maximum ranges consistent with PE, although the minimums are more variable due to the location and depth of the first predicted null.

Table 17 Minimum and maximum ranges (in km) of signal excess for PE, FeyRay, and GRAB for the ASA wedge at 80 dB threshold (maximum detection range is 3.35 km).

Frequency	PE min	FeyRay min	GRAB min	PE max	FeyRay max	GRAB max
25	1.11 km	0.465 km	0.800 km	all	all	all
100	1.96 km	1.895 km	1.005 km	all	all	all
250	0.22 km	1.18 km	0.915 km	all	all	all
2500	0.26 km	0.875 km	2.235 km	all	all	all

Table 18 shows the summary of the correlation error, and average error for FeyRay and GRAB for a 100 dB threshold. The difference in the between the information in table 2 and table 3 is small, as would be expected form inspecting the transmission loss plots. The same conclusions for relative performance drawn from the 80 dB threshold case hold for the 100 dB threshold case.

Table 18 Correlation error and average error for FeyRay and GRAB for an 100 dB threshold for the ASA wedge problem.

Frequency	FeyRay corr error	FeyRay average error	GRAB corr error	GRAB average error
25	40 %	8.7 dB	39 %	3.8 dB
100	43 %	4.2 dB	35 %	1.2 dB
250	45 %	3.4 dB	51 %	1.2 dB
2500	46 %	2.9 dB	51 %	1.1 dB

Table 19. shows the minimum and maximum range of detection for PE, FeyRay, and GRAB for 100 dB threshold. There is no significant difference between the 80 dB threshold case and the 100 dB threshold case.

Table 19 Minimum and maximum ranges (in km) of signal excess for PE, FeyRay, and GRAB for the ASA wedge at 100 dB threshold (maximum detection range is 3.35 km).

Frequency	PE min	FeyRay min	GRAB min	PE max	FeyRay max	GRAB max
25	1.11 km	1.415 km	2.715 km	all	all	all
100	1.96 km	1.895 km	1.005 km	all	all	all
250	0.22 km	1.18 km	0.915 km	all	all	all
2500	0.26 km	0.875 km	2.235 km	all	all	all

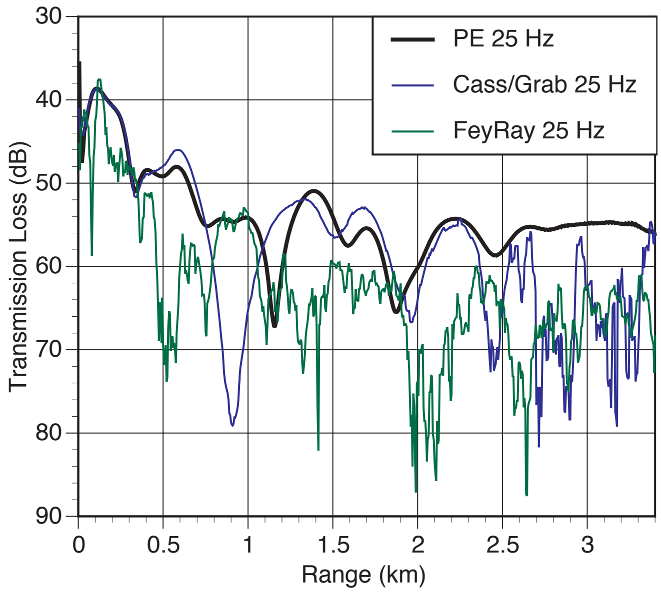

Figure 26 . Plot of transmission loss predictions for PE, FeyRay, and GRAB for ASA wedge problem at a frequency of 25 Hz.

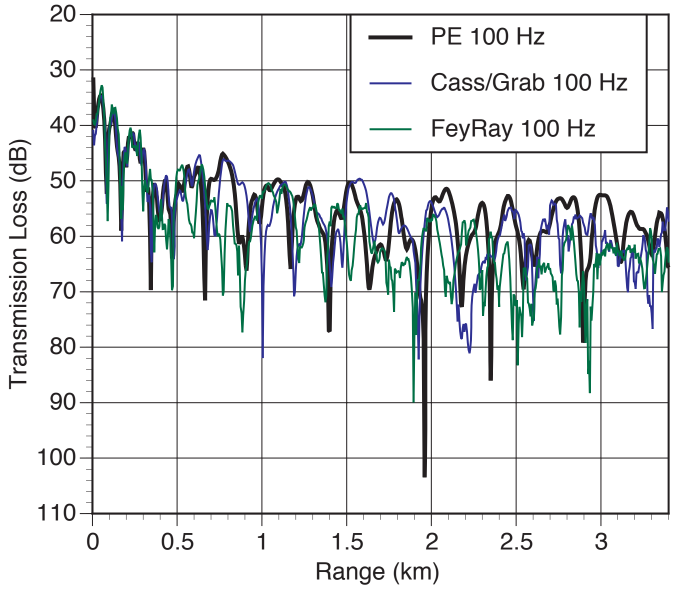

Figure 27 . Plot of transmission loss predictions for PE, FeyRay, and GRAB for ASA wedge problem at a frequency of 100 Hz.

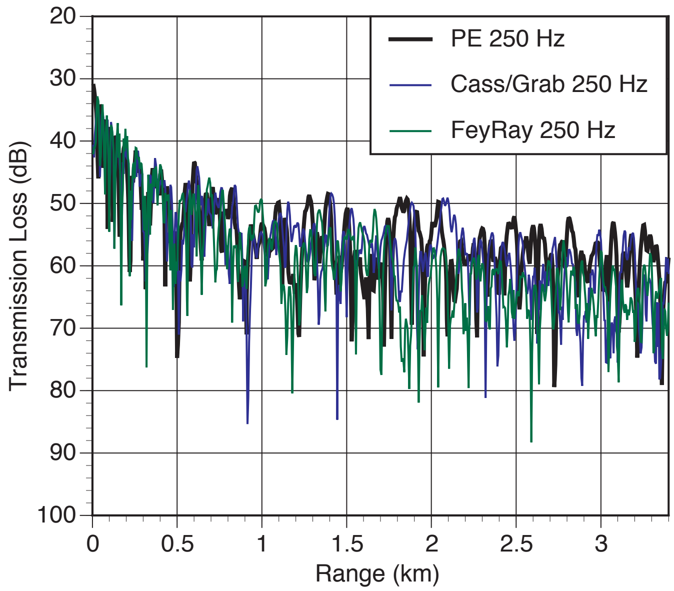

Figure 28 . Plot of transmission loss predictions for PE, FeyRay, and GRAB for ASA wedge problem at a frequency of 250 Hz..

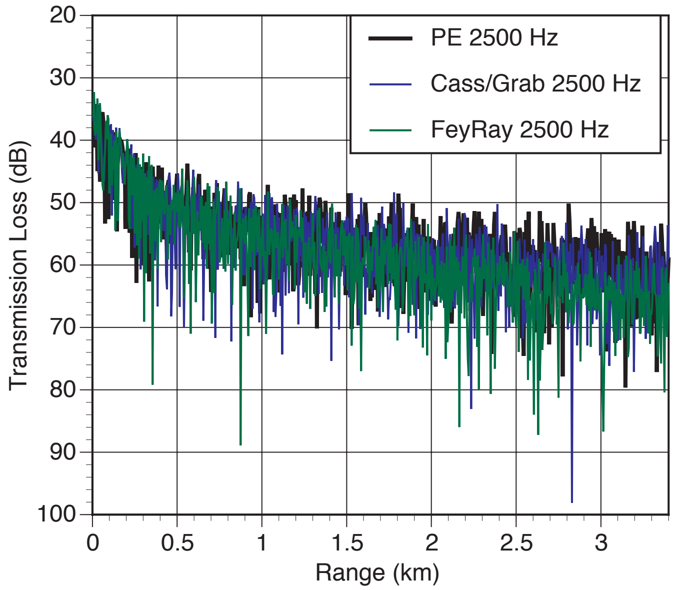

Figure 29 . Plot of transmission loss predictions for PE, FeyRay, and GRAB for ASA wedge problem at a frequency of 2500.

5.2 Upwardly Refracting Range Independent Case.

The upwardly refracting case shares some common aspects with the ASA wedge case. The source, and receive depths are the same. The sound speed is upwardly refracting with a sound speed of 1480 m/s at the surface, and 1500 m/s at the water sediment interface. The water depth is a constant 200 m, and the bottom is a constant reflection loss case. The density is set to 1.5 times that of water, the sound speed ratio is 1, and the attenuation is set to 0.1 dB per wavelength. The bottom condition corresponds to a bottom loss of approximately 13.98 dB per bounce. Each of the models were used to predict transmission loss at the standard frequencies out to a range of 15 km. FeyRay, and GRAB were run using their 'default' ray fans.

Figure 30 shows the PE, FeyRay, and GRAB predictions for the upwardly refracting range independent test case for a frequency of 25 Hz. Both GRAB and FeyRay over-predict the transmission loss. The structure of both predicts is poor. Figure 31 shows the PE, FeyRay and GRAB predictions for the upwardly refracting range independent test case for a frequency of 100 Hz. FeyRay continues to over-predict the transmission loss, while GRAB both over and under predicts the transmission loss. While the overall structure of the transmission loss is still poor for both models, there is a marked improvement over the 25 Hz case. Figure 32 shows the PE, FeyRay and GRAB predictions for the upwardly refracting range independent test case for a frequency of 250 Hz. Both the FeyRay and GRAB predictions are much improved, both in level and in structure. Figure 33 shows the PE, FeyRay and GRAB predictions for the upwardly refracting range independent test case for a frequency of 2500 Hz. Both models generally over-predict the transmission loss, but the overall structure of the loss is much improved from a visual perspective.

Table 20 shows the correlation error, average error, and maximum range of detection for FeyRay, and GRAB for a 60 dB threshold. FeyRay and GRAB are comparable in performance with good correlation error, and low error although the maximum detection range predictions are disappointing.

Table 20 Correlation error, average error, and maximum detection range for FeyRay and GRAB for the flat upwardly refracting test case (maximum detection range is 15 km).

Freq.	FeyRay corr. Err	FeyRay error	GRAB corr err	GRAB error	Max range PE	Max range FeyRay	Max range GRAB
25	18 %	2.1 dB	16 %	2.0 dB	4 km	1 km	1 km
100	15 %	1.9 dB	25 %	2.3 dB	6 km	3 km	1 km
250	14 %	1.7 dB	17 %	1.8 dB	5 km	1 km	2 km
2500	18 %	1.9 dB	18 %	1.9 dB	9 km	5 km	5 km

Table 21 shows the summary of the correlation error and average error for FeyRay and GRAB for a 80 dB threshold. The average error in GRAB is lower than the average error in FeyRay. The correlation error in FeyRay is less at 100 and 250 Hz and greater at 25 and 2500 Hz. Both models, except GRAB at 250 Hz predict more transmission loss than is actually observed.

Table 21. Correlation Error and Average Error for FeyRay and GRAB at 80 dB threshold.

Frequency	FeyRay correlation error	FeyRay average error	GRAB correlation error	GRAB average error
25	27 %	7.9 dB	21 %	5.4 dB
100	27 %	5.1 dB	40 %	5.2 dB
250	37 %	5.2 dB	31 %	-1.5 dB
2500	33 %	5.2 dB	35 %	3.3 dB

Table 22 shows the minimum and maximum range of detection for PE, FeyRay, and GRAB for an 80 dB threshold. FeyRay and GRAB show maximum ranges consistent with PE, although the minimums are more variable due to the location and depth of the first predicted null.

Table 22. Minimum and maximum ranges (in km) of signal excess for PE, FeyRay, and GRAB for the flat upwardly refracting at 80 dB threshold (maximum detection range is 15 km).

Frequency	PE min	FeyRay min	GRAB min	PE max	FeyRay max	GRAB max
25	3.345 km	2.88 km	9.475 km	15 km	14.995 km	14.59 km
100	0.49 km	6.435 km	0.535 km	15 km	14.945 km	15 km
250	1.84 km	1.48 km	14.25 km	15 km	15.0 km	14.995 km
2500	1.84 km	0.96 km	1.49km	15 km	14.885 km	15.0 km

Table 23 shows the summary of the correlation error, and average error for FeyRay and GRAB for a 100 dB threshold. The difference in the between the information in table 2 and table 3 is small, as would be expected form inspecting the transmission loss plots. The same conclusions for relative performance drawn from the 80 dB threshold case hold for the 100 dB threshold case.

Table 23. Correlation Error and Average Error for FeyRay and GRAB at 100 dB threshold.

Frequency	FeyRay corr error	FeyRay average error	GRAB corr error	GRAB average error
25	32 %	9.4 dB	21 %	5.6 dB
100	31 %	5.7 dB	41 %	5.6 dB
250	44 %	5.6 dB	34 %	-1.8 dB
2500	49 %	5.9 dB	42 %	3.5 dB

Table 24. shows the minimum and maximum range of detection for PE, FeyRay, and GRAB for 100 dB threshold. There is no significant difference between the 80 dB threshold case and the 100 dB threshold case. There is tendency for GRAB to over-predict (i.e. range longer than PE prediction) the minimum range at which the signal falls below the threshold.

Table 24. Minimum and maximum ranges (in km) of signal excess for PE, FeyRay, and GRAB for the flat upwardly refracting at 100 dB threshold (maximum detection range is 15 km).

Frequency	PE min	FeyRay min	GRAB min	PE max	FeyRay max	GRAB max
25	3.345 km	3.745 km	9.475 km	15 km	14.945 km	15 km
100	0.49 km	8.74 km	10.58 km	15 km	15 km	15 km
250	14.08 km	8.095 km	14.25 km	15 km	15 km	15 km
2500	4.025 km	7.55 km	9.665 km	15 km	15 km	15 km

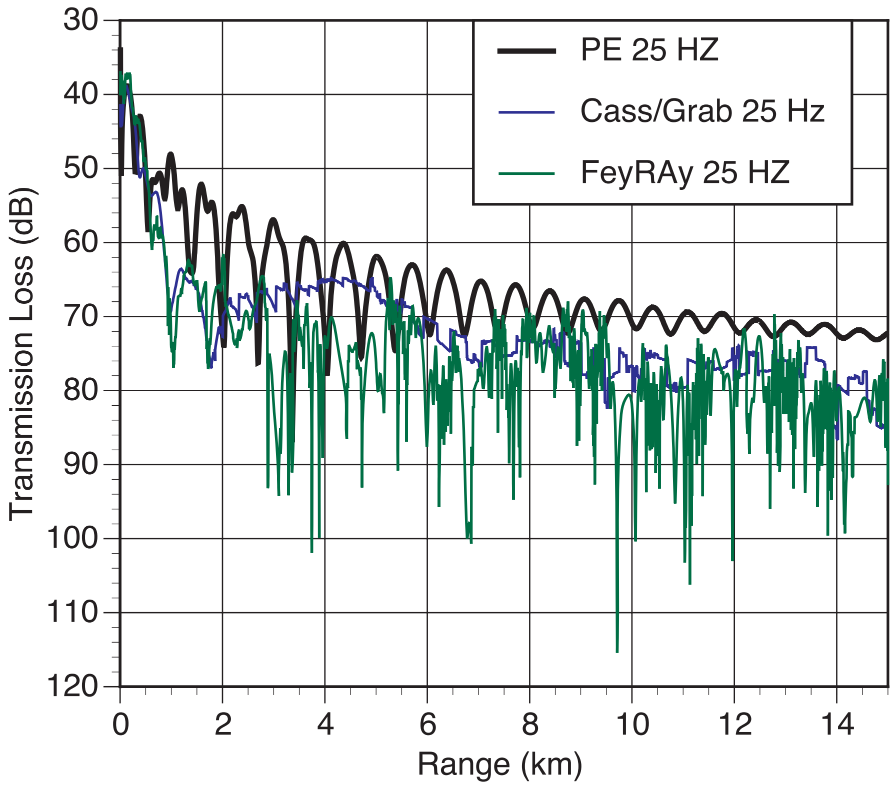

Figure 30 . Plot of transmission loss predictions for PE, FeyRay, and GRAB for the range independent upwardly refracting case at a frequency of 25 Hz.

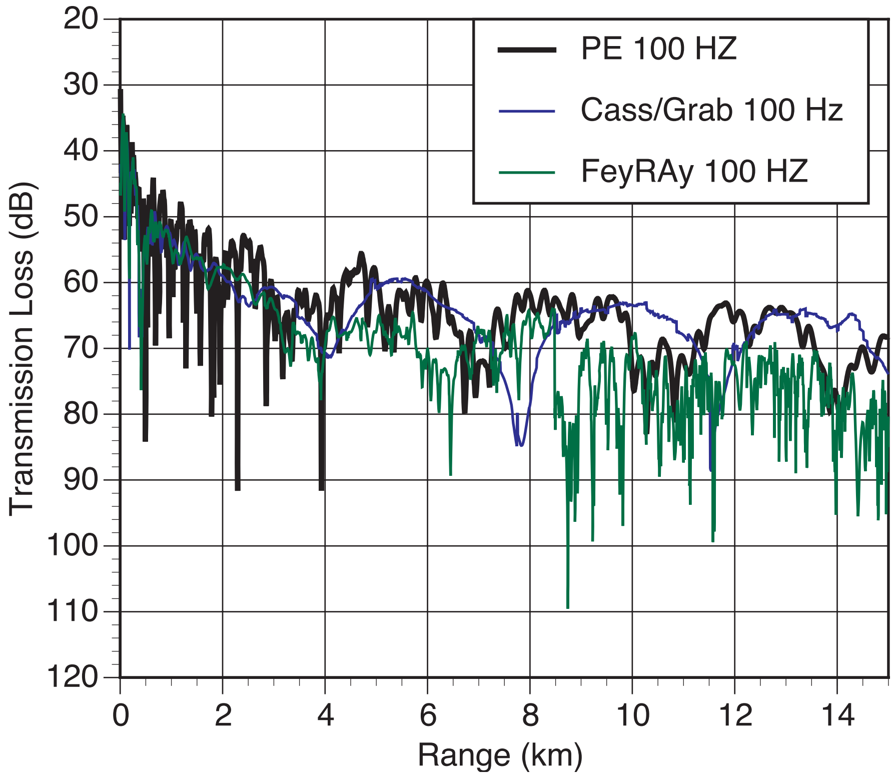

Figure 31 . Plot of transmission loss predictions for PE, FeyRay, and GRAB for the range independent upwardly refracting case at a frequency of 100 Hz.

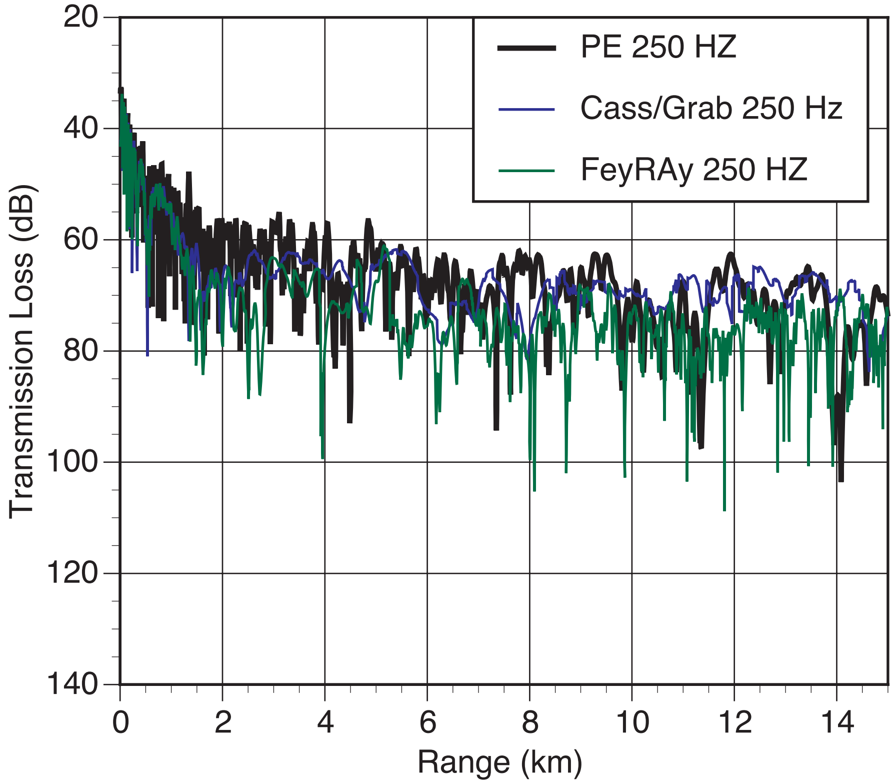

Figure 32 . Plot of transmission loss predictions for PE, FeyRay, and GRAB for the range independent upwardly refracting case at a frequency of 250 Hz.

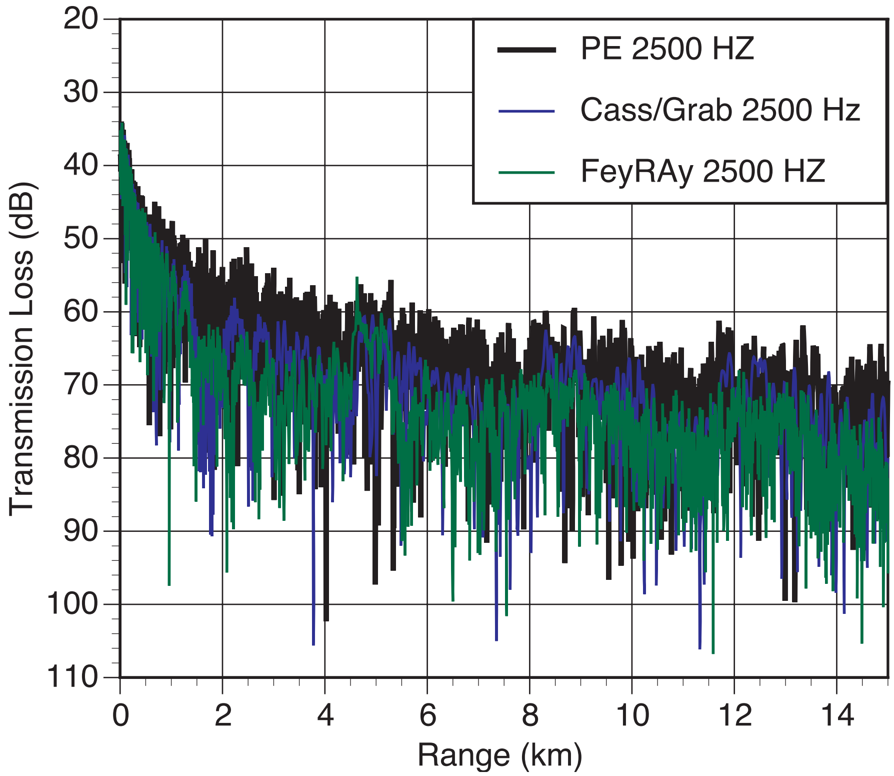

Figure 33 . Plot of transmission loss predictions for PE, FeyRay, and GRAB for the range independent upwardly refracting case at a frequency of 2500 Hz..

5.3 *East China Sea Measured Data*

The East China Sea case is one of two cases (the other being Lloyd's mirror) where the benchmark is not PE. In this case the benchmark is measured transmission loss taken in octave bands ranging from 25 Hz to 800 Hz. The sound speed environment, and bathymetry are depicted in Figure 34. The bottom loss parameters were obtained from inverting the transmission loss data using a simulated annealing method, assuming a two layer bottom. The results were a surface sediment of velocity ratio 1.12, density 1.8, and attenuation 0.4 dB per wavelength of depth 40 m, overlying a sediment of sound speed ratio 1.2, density 2, and attenuation 0.5 dB per wavelength. For these model runs the sediment was set as a Rayleigh bottom with velocity ratio of 1.12, density of 1.8 and attenuation of 0.4 dB per meter. The model output are range averaged over 350 m, and interpolated to the measured data point locations.

Figure 35 shows the measured transmission loss, FeyRay prediction, and GRAB prediction for 25 Hz for the environment depicted in Figure 20. The models follows the measured transmission loss quite well for the entire range, both models exhibit a maximum excursion of 10 dB. Figure 36 shows the measured transmission loss, the FeyRay prediction , and GRAB prediction for 50 Hz for the environment depicted in Figure 20. Again, the predictions are quite reasonable renditions of the measured data.

Figure 37 shows the measured transmission loss, the FeyRay prediction, and GRAB prediction for 100 Hz for the environment depicted in Figure 20. The 100 Hz prediction by GRAB is better than the 100 Hz FeyRay prediction. The GRAB prediction much more closely follows the level of the data. Since the data is largely featureless, tracking features is not an issue. Figure 38 shows the measured transmission loss, FeyRay prediction, and GRAB prediction for 200 Hz. Both models again produce excellent predictions selecting one model over the other based on visual evidence would be difficult. Figure 39 shows the measured transmission loss, the FeyRay prediction, and the GRAB prediction for 400 Hz. Both models

produce very acceptable predictions, with FeyRay visually appearing the more accurate. Figure 40 shows the measured transmission loss, FeyRay prediction, and GRAB prediction for 800 Hz. Both models are under-predict the transmission loss, since the error is systematic some of the difference is probably due to an over-simplistic inversion model. There is a slight upward refracting layer in the sound speed column. Each model fits differently which may account for some or most of the difference

Table 25 shows the correlation error, average error and maximum detection range for FeyRay, and GRAB for a 60 dB threshold, also show is the maximum detection range as measured. The ranges are show in nmi rather than km, as the spacing of the measurement buoys was determined using this measurement system.

Table 25 Correlation error, average error and maximum detection range for FeyRay, and GRAB for the East China Sea Test Case (maximum possible range is 31 nmi).

Freq.	FeyRay corr. Err	FeyRay error	GRAB corr err	GRAB error	Max range measured	Max range FeyRay	Max range GRAB
25	1 %	0.2 dB	3 %	1.4 dB	2 nmi	2 nmi	2 nmi
50	12 %	1.0 dB	2 %	0.9 dB	3 nmi	1 nmi	2 nmi
100	8 %	1.5 dB	5 %	2.6 dB	3 nmi	2 nmi	2 nmi
200	2 %	2.1 dB	10 %	2.6 dB	2 nmi	2 nmi	3 nmi
400	3 %	2.2 dB	3 %	2.6 dB	2 nmi	2 nmi	4 nmi
800	1 %	2.5 dB	7 %	3.3 dB	1 nmi	2 nmi	3 nmi

Table 26 shows the correlation error, and average error for FeyRay and GRAB for a 80 dB threshold. The statistics in this table demonstrate that the visual evidence seen in figures 21 through 26 are reflected in a quantifiable manner. The GRAB correlation error is roughly twice that of the FeyRay correlation error, and the maximum average error for either model is 3.8 dB.

Table 26 Correlation Error and average error for FeyRay and GRAB for the East China Sea measured data case for 80 dB threshold.

Frequency	FeyRay correlation error	FeyRay average error	GRAB correlation error	GRAB average error
25	5 %	1.0 dB	10 %	-1.2 dB
50	6 %	-1.4 dB	10 %	-1.0 dB
100	7 %	-2.5 dB	8 %	-0.2 dB
200	5 %	-1.6 dB	11 %	1.1 dB
400	4%	1.5 dB	10 %	3.1 dB
800	3 %	3.4 dB	6 %	3.8 dB

Table 27 shows the minimum and maximum detection ranges for measured data, FeyRay, and GRAB for a 80 dB threshold. Both models produce acceptable results, which some overestimate of minimum range and some underestimate.

Table 27. Minimum and maximum range of detection for the measured data, FeyRay, and GRAB for the East China Sea test case for 80 dB threshold (maximum detection range is 31 nmi).

Frequency	Measured min	FeyRay min	GRAB min	Measured max	FeyRay max	GRAB max
25	16 nmi	12 nmi	14 nmi	16 nmi	18 nmi	28 nmi
50	18 nmi	9 nmi	13 nmi	18 nmi	18 nmi	18 nmi
100	24 nmi	13 nmi	18 nmi	24 nmi	18 nmi	18 nmi
200	25 nmi	9 nmi	27 nmi44	25 nmi	18 nmi	27 nmi
400	19 nmi	9 nmi	17 nmi	19 nmi	11 nmi	28 nmi
800	11 nmi	9 nmi	14 nmi	11 nmi	11 nmi	27 nmi

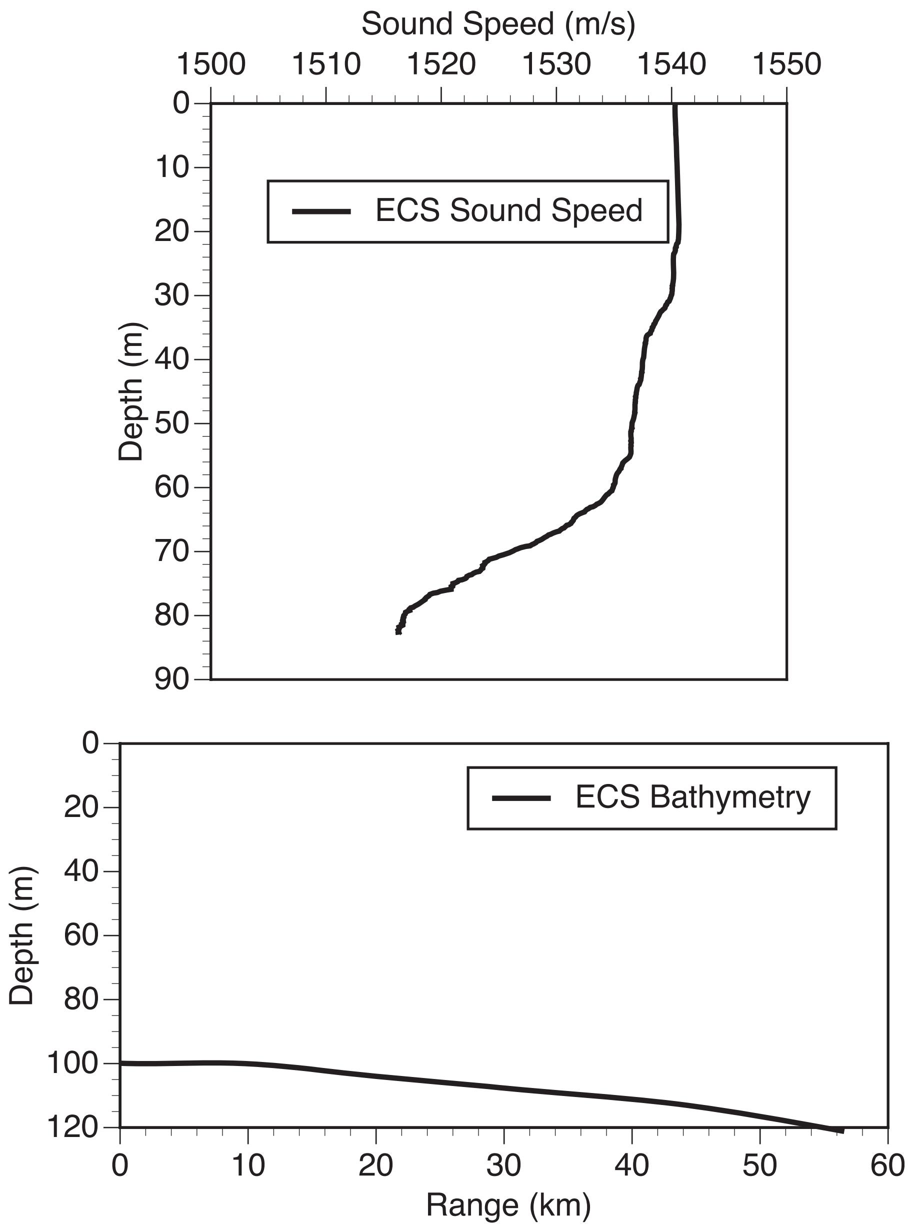

Figure 34 . Plot of the bathymetry, and representative sound speed profile for the East China Sea measured data case.

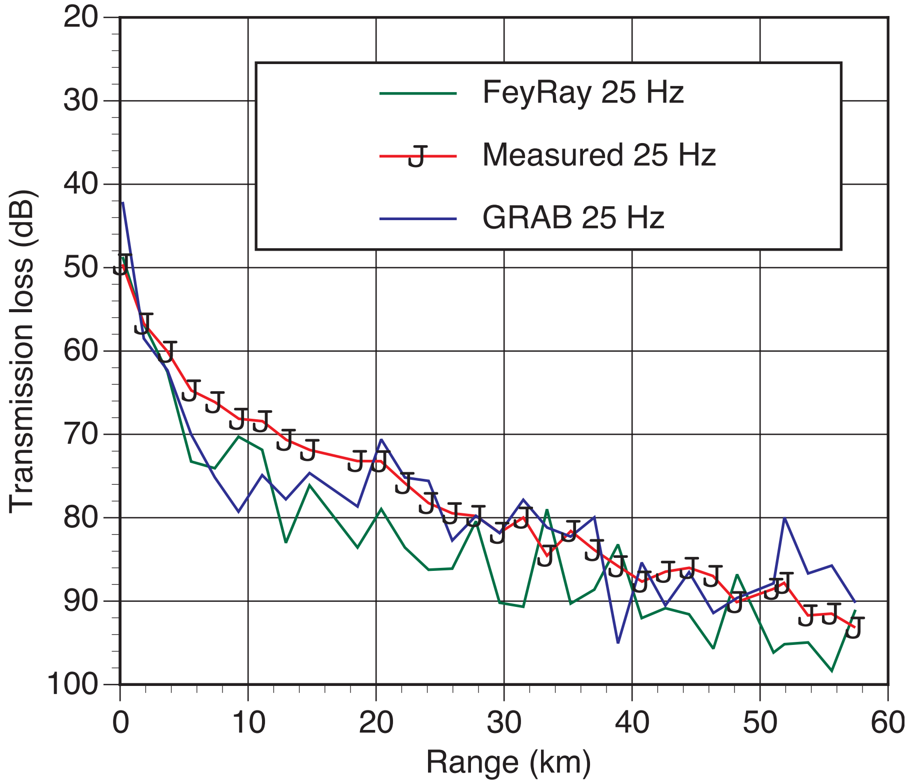

Figure 35 . Plot of transmission loss measured, FeyRay prediction, and GRAB prediction for a frequency of 25 Hz for the East China Sea data case.

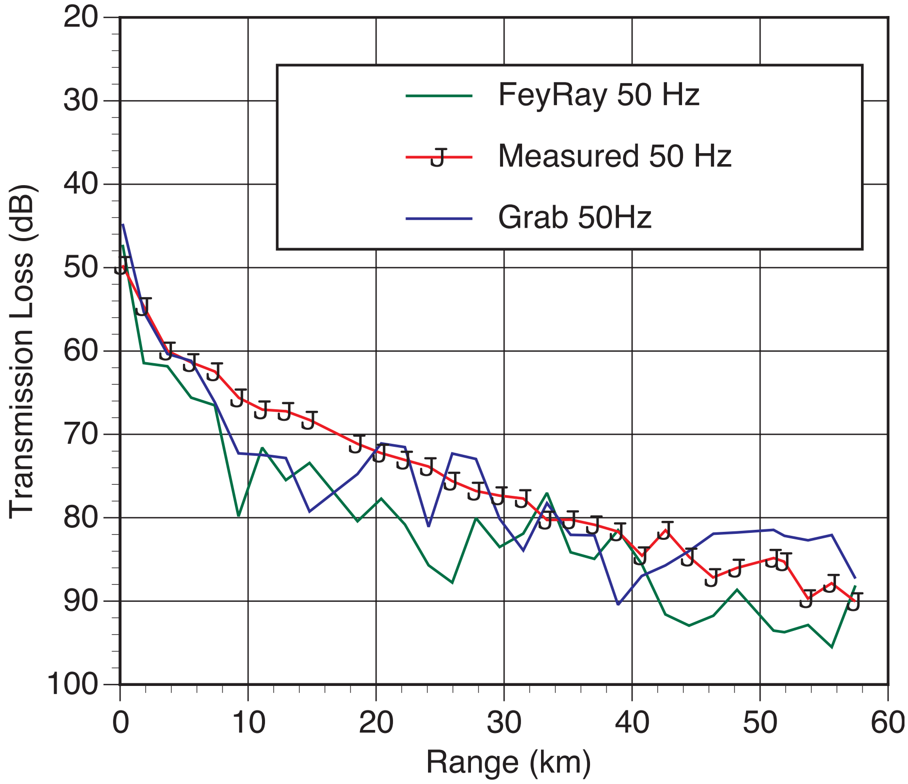

Figure 36 . Plot of transmission loss measured, FeyRay prediction, and GRAB prediction for a frequency of 50 Hz for the East China Sea data.

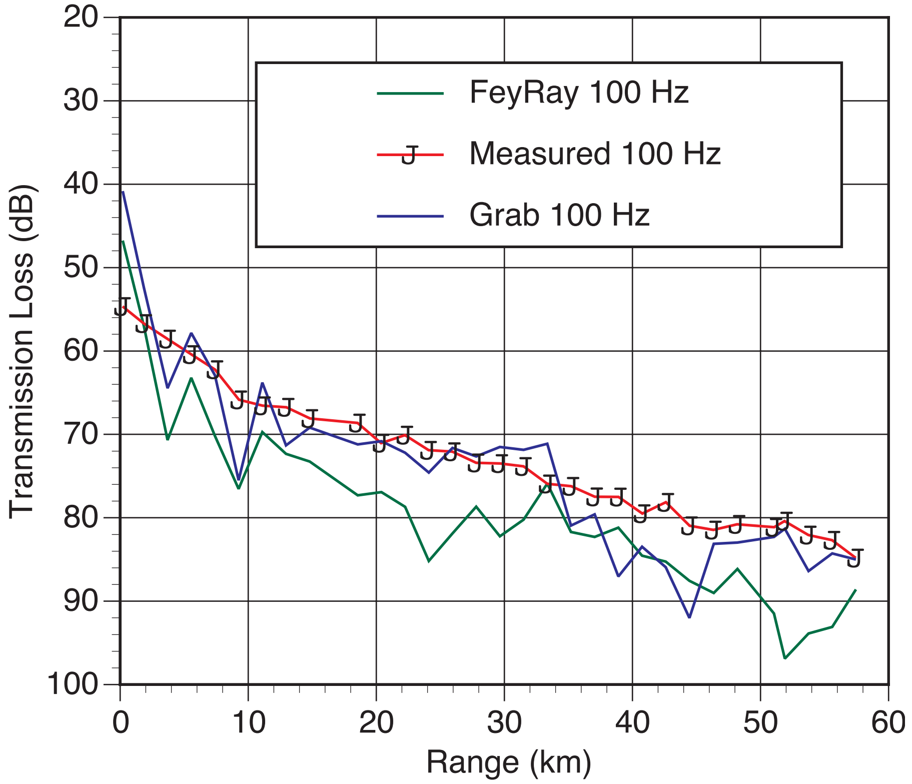

Figure 37 . Plot of transmission loss measured, FeyRay prediction, and GRAB prediction for a frequency of 100 Hz for the East China Sea data.

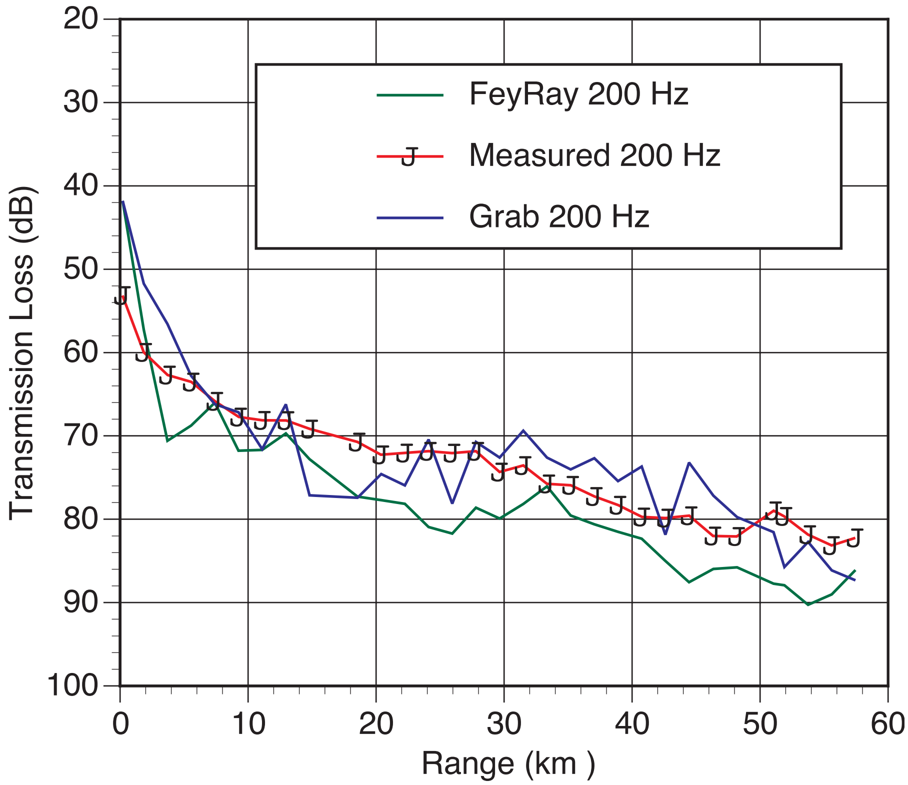

Figure 38 . Plot of transmission loss measured, FeyRay prediction, and GRAB prediction for a frequency of 200 Hz for the East China Sea data case.

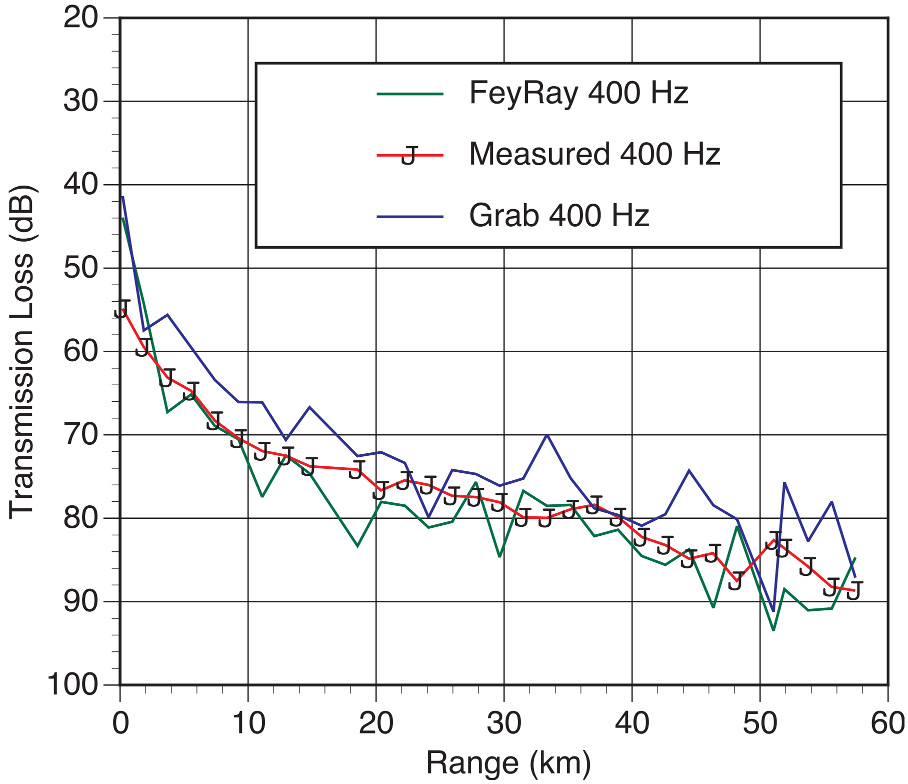

Figure 39 . Plot of transmission loss measured, FeyRay prediction, and GRAB prediction for a frequency of 400 Hz for the East China Sea data case.

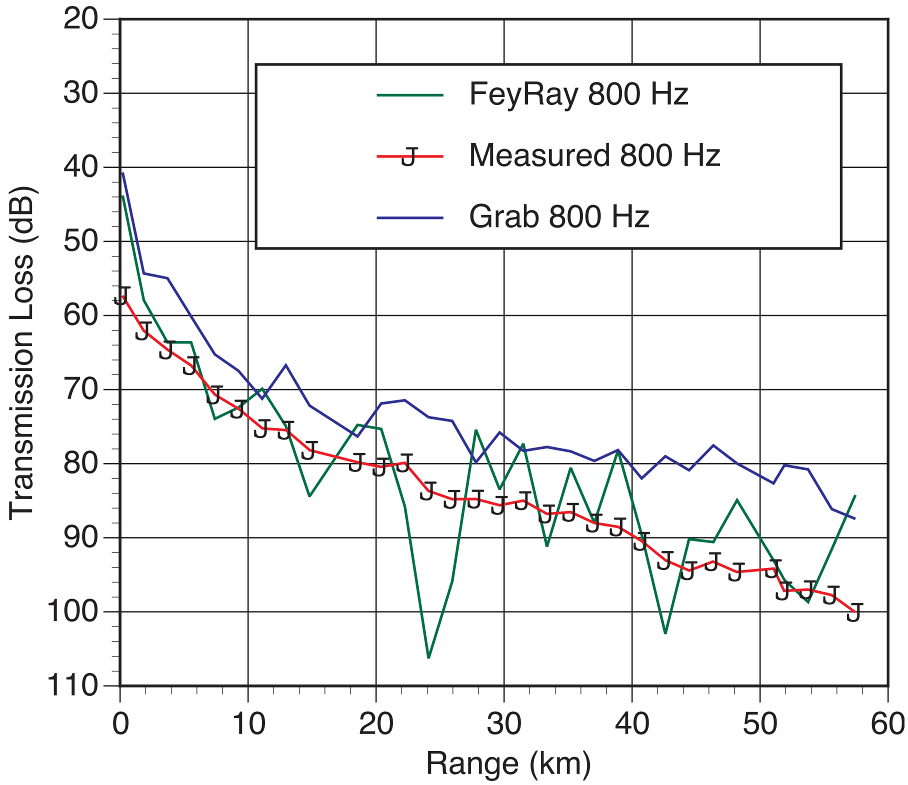

Figure 40 . Plot of transmission loss measured, FeyRay prediction, and GRAB prediction for a frequency of 800 Hz for the East China Sea data case.

5.4 Summary of Shallow Water Cases

Neither FeyRay nor GRAB showed stellar performance against the ASA wedge, or the upwardly refracting range independent test case. In general FeyRay had a better shape, as revealed by the correlation error, and GRAB had a better level. It seems as though GRAB does a better job of predicting the average level over range, but misses the structure of the transmission loss. FeyRay seems to predict the structure better, but the level is slightly worse than GRAB. Against the measured East China Sea data both models performed very well. FeyRay seemed to slightly outperform GRAB in this case, but either model prediction is very good.

6.0 CONCLUSIONS

The deep water and shallow water accuracy test are summarized in Figures 41 and 42. These summaries illustrate the average error values across frequencies for the 100 dB threshold cases. Figure 41 represents the mean and standard deviation of the “average difference in level” metric. Figure 42 represents the mean and standard deviation of the “average correlation error” metric. The results are presented as a bar chart in which the performance of GRAB and FeyRay, relative to a reference solution (NSPE or sea test measurements), is illustrated for each of the deep water and shallow water accuracy tests.

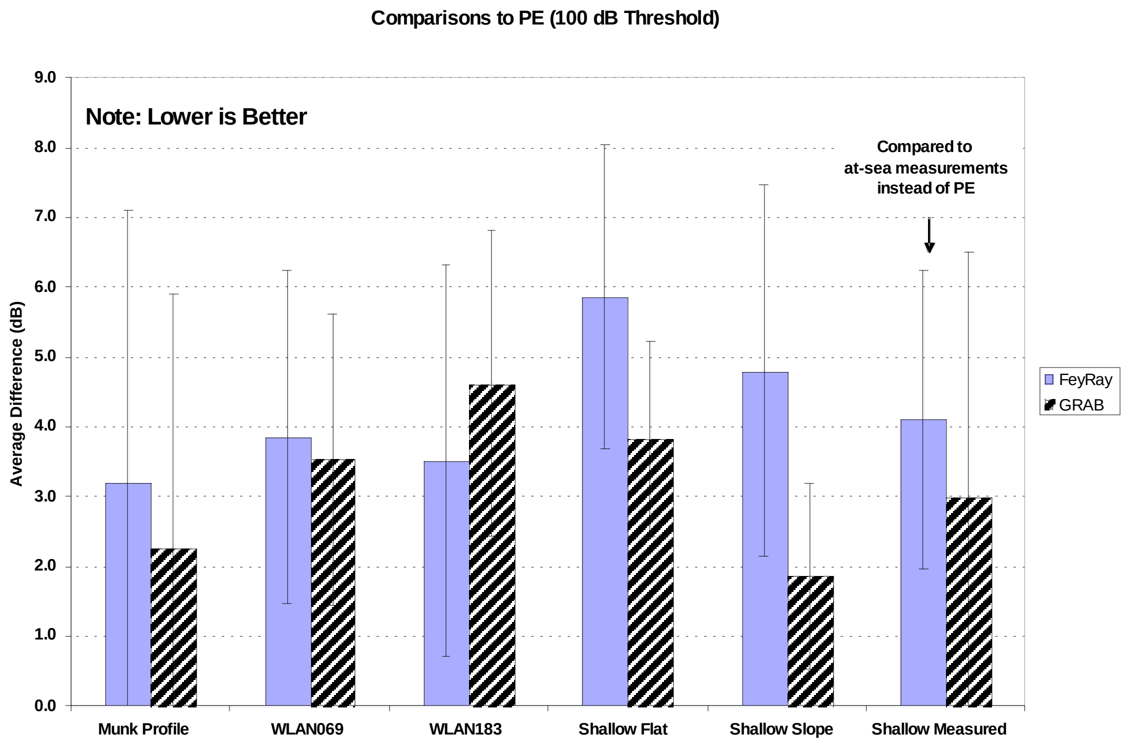

Figure 41 . Plot of transmission loss measured, FeyRay prediction, and GRAB prediction for a frequency of 800 Hz for the East China Sea data case.

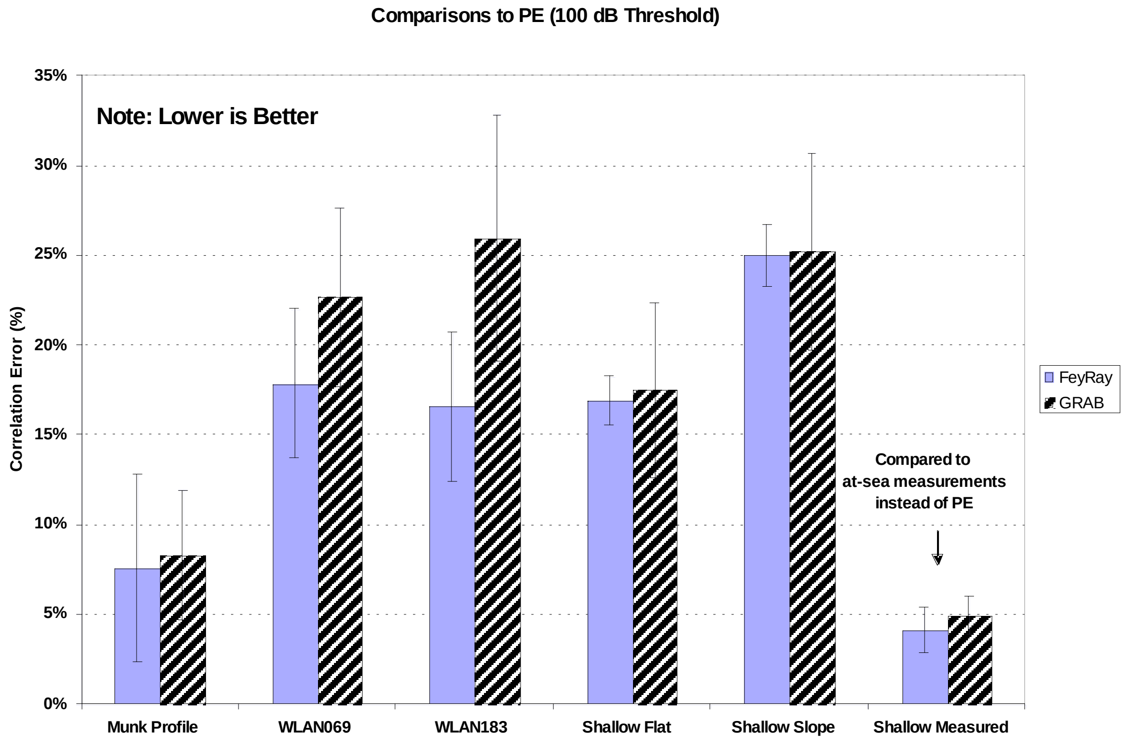

Figure 42 . Plot of transmission loss measured, FeyRay prediction, and GRAB prediction for a frequency of 800 Hz for the East China Sea data case.

In general, the accuracy tests show that the level of GRAB is slightly closer to the reference than the corresponding FeyRay results. But, the correlation error of the FeyRay results are slightly lower than their GRAB counterparts. However, the error bars in Figures 41 and 42 indicate that these differences are not statistically significant. Figures 4-11 illustrate that a similar conclusion of equivalence can be drawn for the random environments sampled in the robustness tests. According to these measurements, the propagation loss values computed by FeyRay are statistically equivalent in both level and structure to the values computed by the GRAB model.

The timing tests reveal that the execution speed of FeyRay is significantly better than that of GRAB. Even after eliminating differences in the time needed to load the program into memory and GRAB's heavier use of disk I/O, the FeyRay calculations ran approximately 20 times faster than GRAB results of the same accuracy. Recent changes have been made to GRAB to improve its speed (at the cost of accuracy), but FeyRay was still 5 times faster than the improved GRAB model.

In my judgment, these measurements indicate that the accuracy and execution speed of FeyRay are sufficient to justify its use in high-fidelity training systems.

7.0 REFERENCES

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